MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

The Rev. John Turnbull Bird, M.A., Chaplain to the Forces, care of Sir C. R. M'Grigor, Bart., & Co., 25 Charles Street, St. James's Square, London, S.W.;
Charles Frederick William Halliley, Badulla, Ceylon, and Junior Conservative Club, 43 Albemarle Street, London, W.;
Ernest W. Hardy, F.R.G.S., Head Master, Westing Schoolhouse, Uyea Sound, Unst, Shetland, N.B.; and
Friedrich Wilhelm Ristenpart, Ph.D., Director of the Observatory, Santiago de Chile,

were balloted for and duly elected Fellows of the Society.

Professor Hermann Albert Kobold, Director of the Centralstelle, Kiel, Germany,

was balloted for and duly elected an Associate of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

His Highness Raj Rana Bhawani Singh Bahadur of Jhalawar, Rajputana, Vice-President of the Astronomical Society of India, Jhalrapatan, India (proposed by Sir David Gill);
Thomas John I'Anson Bromwich, Sc.D., F.R.S., St. John's College, Cambridge (proposed by F. J. M. Stratton);
William Shiach Cumming, H.M. Civil Service (Revenue De-

partment), Craig Lynn, Rothes, Morayshire (proposed by the Rev. Charles Whyte); and

Donald Maclean, Coats Observatory, Paisley (proposed by F. W. Dyson).

Eighty presents were announced as having been received since the last meeting, including, amongst others:—

Greenwich Observations, 1910, presented by the Royal Observatory, Greenwich; Perth Observatory, Western Australia, Astrographic Catalogue, Perth Section, vols. 1 and 4; and Catalogue of 2043 stars, presented by the Observatory.

Three photographs of the solar eclipse of 1912 April 16–17, presented by E. C. Willis.

On Librating Planets and on a New Family of Periodic Orbits.

By Sir George Darwin, K.C.B., F.R.S.

§ 1. *Librating Planets.*

In Professor Ernest Brown's interesting paper on "A New Family of Periodic Orbits" (*M.N.*, *R.A.S.*, vol. lxxi., 1911, p. 438) he shows how to obtain the orbit of a planet which makes large oscillations about the vertex of the Lagrangian equilateral triangle. In discussing this paper I shall depart slightly from his notation, and use that of my own paper on "Periodic Orbits" (*Scientific Papers*, vol. iv., or *Acta Math.*, vol. li.). "Jove," J, of mass 1, revolves at distance 1 about the Sun, S, of mass ν, and the orbital angular velocity is n, where $n^2 = \nu + 1$.

The axes of reference revolve with SJ as axis of x, and the heliocentric and jovicentric rectangular co-ordinates of the third body are x, y and $x-1, y$ respectively. The heliocentric and jovicentric polar co-ordinates are respectively r, θ and ρ, ψ. The potential function for relative energy is Ω.

The equations of motion and Jacobian integral, from which Brown proceeds, are

$$\left.\begin{aligned} \frac{d^2r}{dt^2} - r\frac{d\theta}{dt}\left(\frac{d\theta}{dt} + 2n\right) &= \frac{\partial\Omega}{\partial r}, \\ \frac{d}{dt}\left[r^2\left(\frac{d\theta}{dt} + n\right)\right] &= \frac{\partial\Omega}{\partial\theta}, \\ \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2 &= 2\Omega - C, \end{aligned}\right\} \quad . \quad (1)$$

where

$$2\Omega = \nu\left(r^2 + \frac{2}{r}\right) + \left(\rho^2 + \frac{2}{\rho}\right).$$

The following are rigorous transformations derived from these equations, virtually given by Brown in approximate forms in equation (13), and at the foot of p. 443 :—

$$\left(\frac{d\theta}{dt}+n\right)^2 = A+\frac{1}{r}\frac{d^2r}{dt^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{dr}{dt}\left(L+3\frac{d^2r}{dt^2}\right) = B+D\left(\frac{d\theta}{dt}+n\right)-r\frac{d^3r}{dt^3}, \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{d^2r}{dt^2}\left(L+3\frac{d^2r}{dt^2}\right) = E\left(\frac{dr}{dt}\right)^2+F\frac{dr}{dt}\frac{d\theta}{dt}+G\left(\frac{d\theta}{dt}\right)^2+H\frac{dr}{dt}+J\frac{d\theta}{dt}+K$$
$$-4\frac{dr}{dt}\frac{d^3r}{dt^3}-r\frac{d^4r}{dt^4}, \quad (4)$$

where

$$A = n^2-\frac{\partial\Omega}{r\partial r} = \frac{\nu}{r^3}+1-\frac{1}{r}\left(\rho-\frac{1}{\rho^2}\right)\cos(\theta-\psi),$$

$$B = -nr\frac{\partial^2\Omega}{r\partial r\partial\theta} = -n\sin\psi\left[\left(\rho-\frac{1}{\rho^2}\right)+\frac{3r}{\rho^3}\cos(\theta-\psi)\right],$$

$$D = r\frac{\partial^2\Omega}{\partial r\partial\theta}+2\frac{\partial\Omega}{\partial\theta} = 3\sin\psi\left[\left(\rho-\frac{1}{\rho^2}\right)+\frac{r}{\rho^3}\cos(\theta-\psi)\right],$$

$$L = 4n^2r-r\frac{\partial^2\Omega}{\partial r^2}-3\frac{\partial\Omega}{\partial r} = \frac{\nu}{r^2}+3r+\frac{r}{\rho^3}-3\left(\rho-\frac{1}{\rho^2}\right)\cos(\theta-\psi)$$
$$-\frac{3r}{\rho^3}\cos^2(\theta-\psi),$$

$$E = r\frac{\partial^3\Omega}{\partial r^3}+4\frac{\partial^2\Omega}{\partial r^2}-4n^2 = \frac{2\nu}{r^3}+\frac{4}{\rho^3}[3\cos^2(\theta-\psi)-1]$$
$$+\frac{3r}{\rho^4}\cos(\theta-\psi)[3-5\cos^2(\theta-\psi)],$$

$$F = 2r\frac{\partial^3\Omega}{\partial r^2\partial\theta}+4\frac{\partial^2\Omega}{\partial r\partial\theta}-4\frac{\partial\Omega}{r\partial\theta} = \frac{6}{\rho^4}\sin\psi[5r\sin^2(\theta-\psi)-4\cos\theta],$$

$$G = r\frac{\partial^3\Omega}{\partial r\partial\theta^2}+2\frac{\partial^2\Omega}{\partial\theta^2} = \frac{3r}{\rho}\left[\left(\rho-\frac{1}{\rho^2}\right)\cos\theta-\frac{r}{\rho^3}\sin\psi(5\sin^2(\theta-\psi)-1)\right],$$

$$H = -\frac{4n}{r}\frac{\partial\Omega}{\partial\theta} = 4n\left(\rho-\frac{1}{\rho^2}\right)\sin(\theta-\psi),$$

$$J = 2n\frac{\partial^2\Omega}{\partial\theta^2} = \frac{2nr}{\rho}\left[\left(\rho-\frac{1}{\rho^2}\right)\cos\theta-\frac{3}{\rho^2}\sin\psi\sin(\theta-\psi)\right],$$

$$K = \frac{\partial\Omega}{r^2\partial\theta}\left(r\frac{\partial^2\Omega}{\partial r\partial\theta}+2\frac{\partial\Omega}{\partial\theta}\right) = \frac{3}{r}\left(\rho-\frac{1}{\rho^2}\right)\sin\theta\sin\psi\left(1+\frac{1}{\rho^4}\cos\psi\right).$$

A great diversity of forms might be given to these functions, but the foregoing seemed to be as convenient for computation as I could devise.

It is known that when ν is less than 24·9599 * the vertex of

* "Periodic Orbits," *Scientific Papers*, vol. iv. p. 73.

the equilateral triangle is an unstable solution of the problem, and if the body is displaced from the vertex it will move away in a spiral orbit. Hence for small values of ν there are no small closed periodic orbits of the kind considered by Brown. But certain considerations led him to conjecture that there might still exist large oscillations of this kind. The verification of such a conjecture would be interesting, and in my attempt to test his idea I took ν equal to 10. This value was chosen because the results will thus form a contribution towards that survey of periodic orbits which I have made in previous papers for ν equal to 10.

Brown's system of approximation, which he justifies for large values of ν, may be described, as far as material for my present object, as follows:—

We begin the operation at any given point r, θ, such that ρ is greater than unity.

Then in (2) and (3) $\frac{d^2r}{dt^2}$ and $\frac{d^3r}{dt^3}$ are neglected, and we thence find $\frac{dr}{dt}$, $\frac{d\theta}{dt}$.

By means of these values of the first differentials, and neglecting $\frac{d^3r}{dt^3}$ and $\frac{d^4r}{dt^4}$ in (4), we find $\frac{d^2r}{dt^2}$ from (4).

Returning to (2) and (3) and using this value of $\frac{d^2r}{dt^2}$, we redetermine the first differentials, and repeat the process until the final values of $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ remain unchanged. We thus obtain the velocity at this point r, θ on the supposition that $\frac{d^3r}{dt^3}$, $\frac{d^4r}{dt^4}$ are negligible, and on substitution in the last of (1) we obtain the value of C corresponding to the orbit which passes through the chosen point.

Brown then shows how the remainder of the orbit may be traced with all desirable accuracy in the case where ν is large. It does not concern me to follow him here, since his process could scarcely be applicable for small values of ν. But if his scheme should still lead to the required result, the remainder of the orbit might be traced by quadratures, and this is the plan which I have adopted. If the orbit as so determined proves to be clearly non-periodic, it seems safe to conclude that no widely librating planets can exist for small values of ν.

I had already become fairly confident from a number of trials, which will be referred to hereafter, that such orbits do not exist; but it seemed worth while to make one more attempt by Brown's procedure, and the result appears to be of sufficient interest to be worthy of record.

For certain reasons I chose as my starting-point

$$x_0 = -\cdot 36200, \quad y_0 = \cdot 93441,$$

which give

$$r_0 = 1{\cdot}00205, \quad \rho_0 = 1{\cdot}65173.$$

The successive approximations to C were found to be

$$33{\cdot}6977, \quad 33{\cdot}7285, \quad 33{\cdot}7237, \quad 33{\cdot}7246, \quad 33{\cdot}7243.$$

I therefore took the last value as that of C, and found also that

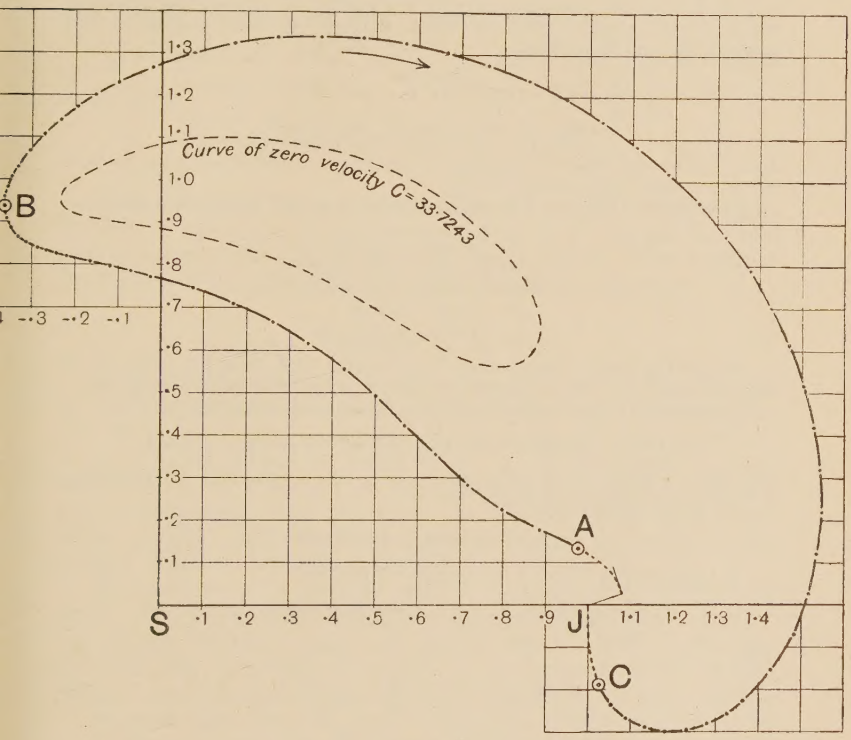


FIG. 1.—Results derived from Professor Brown's Method.

the direction of motion was given by $\phi_0 = 2^\circ 21'$. These values of x_0, y_0, ϕ_0, and C then furnish the values from which to begin the quadratures.

Fig. 1 shows the result, the starting-point being at B. The curve was traced backwards to A and onwards to C, and the computed positions are shown by dots connected into a sweeping curve by dashes.

From A back to perijove and from C on to J the orbit

was computed as undisturbed by the Sun.* Within the limits of accuracy adopted the body collides with J.

Since the curve comes down on to the negative side of the

* When the body has been traced to the neighbourhood of J, let it be required to determine its future position on the supposition that the solar perturbation is negligible. Since the axes of reference are rotating, the solution needs care, and it may save the reader some trouble if I set down how it may be done conveniently.

Let the co-ordinates, direction of motion, and velocity, at the moment $t=0$ when solar perturbation is to be neglected, be given by x_0, y_0 (or r_0, θ_0, and ρ_0, ψ_0), ϕ_0, V_0; and generally let the suffix 0 to any symbol denote its value at this epoch. Then the mean distance a, mean motion μ, and eccentricity e are found from

$$\frac{1}{a}=\frac{2}{\rho_0}-[V_0^2+2n\rho_0V_0\cos(\phi_0-\psi_0)+n^2\rho_0^2],\quad \mu^2a^3=1$$
$$a(1-e^2)=[V_0\rho_0\cos(\phi_0-\psi_0)+n\rho_0^2]^2.$$

Let $t=\tau$ be the time of passage of perijove, so that when τ is positive perijove is later than the epoch $t=0$.

At any time t let ρ, v, E be radius vector, true and eccentric anomalies; then

$$\rho=a(1-e\cos E),$$
$$\rho^{\frac{1}{2}}\cos\tfrac{1}{2}v=a^{\frac{1}{2}}(1-e)^{\frac{1}{2}}\cos\tfrac{1}{2}E,$$
$$\rho^{\frac{1}{2}}\sin\tfrac{1}{2}v=a^{\frac{1}{2}}(1+e)^{\frac{1}{2}}\sin\tfrac{1}{2}E,$$
$$\mu(t-\tau)=E-e\sin E,$$
$$\psi=\psi_0-v_0+v-nt.$$

On putting $t=0$, E_0 and τ may be computed from these formulæ, and it must be noted that when τ is positive E_0 and v_0 are to be taken as negative.

The position of the body as it passes perijove is clearly given by

$$x-1=a(1-e)\cos(\psi_0-v_0-n\tau),\quad y=a(1-e)\sin(\psi_0-v_0-n\tau).$$

Any other position is to be found by assuming a value for E, computing ρ, v, t, ψ, and using the formulæ

$$x-1=\rho\cos\psi,\quad y=\rho\sin\psi.$$

In order to find V and ϕ we require the formulæ

$$\frac{1}{\rho}\frac{d\rho}{dt}=\frac{ae\sin E}{\rho}\cdot\frac{\mu a}{\rho};\qquad \frac{dv}{dt}=\frac{[a(1-e^2)]^{\frac{1}{2}}}{\rho}\cdot\frac{a^{\frac{1}{2}}}{\rho}\cdot\frac{\mu a}{\rho},$$

and

$$V\sin\phi=-\frac{(x-1)}{\rho}\frac{d\rho}{dt}+y\left(\frac{dv}{dt}-n\right),$$
$$V\cos\phi=\frac{y}{\rho}\frac{d\rho}{dt}+(x-1)\left(\frac{dv}{dt}-n\right).$$

The value of V as computed from these should be compared with that derived from

$$V^2=\nu\left(r^2+\frac{2}{r}\right)+\left(\rho^2+\frac{2}{\rho}\right)-C,$$

and if the two agree pretty closely, the assumption as to the insignificance of solar perturbation is justified.

If the orbit is retrograde about J, care has to be taken to use the signs correctly, for v and E will be measured in a retrograde direction, whereas ψ will be measured in the positive direction.

A similar investigation is applicable, *mutatis mutandis*, when the body passes very close to *S*.

line of syzygy SJ it differs much from Brown's orbits, and it is clear that it is not periodic. Thus his method fails, and there is good reason to believe that his conjecture is unfounded.

After this work had been done Professor Brown pointed out to me in a letter that if his process be translated into rectangular co-ordinates, the approximate expressions for dx/dt and dy/dt will have as a divisor the function

$$Q = \left(4n^2 - \frac{\partial^2\Omega}{\partial x^2}\right)\left(4n^2 - \frac{\partial^2\Omega}{\partial y^2}\right) - \left(\frac{\partial^2\Omega}{\partial x \partial y}\right)^2.$$

The method will then fail if Q vanishes or is small.

I find that if we write $\Gamma = \frac{\nu}{r^3} + \frac{1}{\rho^3}$, the divisor may be written in the form

$$Q = (3n^2 + \Gamma)(3n^2 - 2\Gamma) + \frac{\rho\nu}{r^5\rho^5}\sin\theta\sin\psi\,.$$

Now, Mr. T. H. Brown, Professor Brown's pupil, has traced one portion of the curve $Q = 0$, corresponding to $\nu = 10$, and he finds that it passes rather near to the orbit I have traced. This confirms the failure of the method which I had concluded otherwise.

§ 2. *Variation of an Orbit.*

A great difficulty in determining the orbits of librating planets by quadratures arises from the fact that these orbits do not cut the line of syzygy at right angles, and therefore the direction of motion is quite indeterminate at every point. I endeavoured to meet this difficulty by a method of variation which is certainly feasible, but, unfortunately, very laborious. In my earlier attempts I had drawn certain orbits, and I attempted to utilise the work by the method which will now be described.

The stability of a periodic orbit is determined by varying the orbit. The form of the differential equation which the variation must satisfy does not depend on the fact that the orbit is periodic, and thus the investigation in §§ 8, 9 of my paper on "Periodic Orbits" remains equally true when the varied orbit is not periodic.

Suppose, then, that the body is displaced from a given point of a non-periodic orbit through small distances $\delta q V^{\frac{1}{2}}$ along the outward normal and δs along the positive tangent, then we must have

$$\frac{d^2\delta q}{ds^2} + \Psi\delta q = 0\,,$$

$$\frac{d}{ds}\left(\frac{\delta s}{V}\right) = -\frac{2\delta q}{V^{\frac{3}{2}}}\left(\frac{1}{R} + \frac{n}{V}\right),$$

where

$$\Psi = \frac{5}{2}\left(\frac{1}{R} + \frac{n}{V}\right)^2 - \frac{3}{2V^2}\left[\frac{\nu}{r^3}\cos^2(\phi - \theta) + \frac{1}{\rho^3}\cos^2(\phi - \psi)\right] + \frac{3}{4}\left(\frac{dV}{Vds}\right)^2,$$

and

$$\frac{dV}{Vds} = \frac{\nu}{V^2}\left(\frac{1}{r^2} - r\right)\sin(\phi - \theta) + \frac{1}{V^2}\left(\frac{1}{\rho^2} - \rho\right)\sin(\phi - \psi).$$

Also

$$\delta\phi = -\frac{1}{V^4}\left[\frac{d\delta q}{ds} - \tfrac{1}{2}\delta q\left(\frac{dV}{Vds}\right)\right] + \frac{\delta s}{R}.$$

Since it is supposed that the co-ordinates, direction of motion, and radius of curvature R have been found at a number of points equally distributed along the orbit, it is clear that Ψ may be computed for each of those points.

At the point chosen as the starting-point the variation may be of two kinds:—

(1) $\delta q_0 = a$, $\quad \dfrac{d\delta q_0}{ds} = 0$, where a is a constant,

(2) $\delta q_0 = 0$, $\quad \dfrac{d\delta q_0}{ds} = b$, where b is a constant.

Each of these will give rise to an independent solution, and if in either of them a or b is multiplied by any factor, that factor will multiply all the succeeding results. It follows, therefore, that we need not concern ourselves with the exact numerical values of a or b, but the two solutions will give us all the variations possible. In the first solution we start parallel with the original curve at the chosen point on either side of it, and at any arbitrarily chosen small distance. In the second we start from the chosen point, but at any arbitrary small inclination on either side of the original tangent.

The solution of the equations for δq and δs have to be carried out step by step along the curve, and it may be worth while to indicate how the work may be arranged.

The length of arc from point to point of the unvaried orbit may be denoted by Δs, and we may take four successive values of Ψ, say, Ψ_{n-1}, Ψ_n, Ψ_{n+1}, Ψ_{n+2}, as affording a sufficient representation of the march of the function Ψ throughout the arc Δs between the points indicated by n to $n+1$.

If the differential equation for δq be multiplied by $(\Delta s)^2$, and if we introduce a new independent variable z such that $dz = ds/\Delta s$, and write $X = \Psi(\Delta s)^2$, the equation becomes

$$\frac{d^2\delta q}{dz^2} = -X\delta q,$$

and z increases by unity as the arc increases by Δs.

Suppose that the integration has been carried as far as the point n, and that δq_0, $d\delta q_0/dz$ are the values at that point; then it is required to find δq_1, $d\delta q_1/dz$ at the point $n+1$.

If the four adjacent values of X are X_{-1}, X_0, X_1, X_2, and if

$\delta = X_1 - X_0$, $\delta^2 = \frac{1}{2}[(X_2 - 2X_1 + X_0) + (X_1 - 2X_0 + X_{-1})]$, Bessel's formula for the function X is

$$X = X_0 + (\delta - \tfrac{1}{2}\delta^2)z + \tfrac{1}{2}\delta^2 z^2 .$$

We now assume that throughout the arc n to $n+1$,

$$\delta q = \delta q_0 + \frac{d\delta q_0}{dz} z + Q_2 z^2 + Q_3 z^3 + Q_4 z^4 ,$$

where Q_2, Q_3, Q_4 have to be determined so as to satisfy the differential equation.

On forming the product $X\delta q$, integrating, and equating coefficients, we find $Q_2 = -\frac{1}{2}X_0\delta q_0$, and the values of Q_3, Q_4, are easily found. In carrying out this work I neglect all terms of the second order except X_0^2.

The result may be arranged as follows :—

Let

$$A = 1 - \tfrac{1}{2}X_0 - \tfrac{1}{6}\delta + \tfrac{1}{24}(\delta^2 + X_0^2),$$
$$B = 1 - \tfrac{1}{6}X_0 - \tfrac{1}{12}\delta + \tfrac{1}{24}\delta^2,$$
$$C = X_0 + \tfrac{1}{2}\delta + \tfrac{1}{12}\delta^2 - \tfrac{1}{6}(\delta^2 + X_0^2),$$
$$D = 1 - \tfrac{1}{2}X_0 - \tfrac{1}{3}\delta + \tfrac{1}{6}\delta^2 ;$$

then, on putting $z = 1$, we find

$$\delta q_1 = A\delta q_0 + B\frac{d\delta q_0}{dz},$$

$$\frac{d\delta q_1}{dz} = -C\delta q_0 + D\frac{d\delta q_0}{dz}.$$

When the Ψ's have been computed, the X's and A, B, C, D are easily found at each point of the unvaried orbit. We then begin the two solutions from the chosen starting-point, and thus trace δq and $d\delta q/dz$ from point to point both backwards and forwards. The necessary change of procedure when Δs changes in magnitude is obvious.

The procedure is tedious although easy, but the work is enormously increased when we pass on further to obtain an intelligible result from the integration. When δq and $d\delta q/dz$ have been found at each point, a further integration has to be made to determine δs, and this has, of course, to be done for each of the solutions. Next, we have to find the normal displacement δp (equal to $\delta q V^{-\frac{1}{2}}$), and, finally, δp, δs have to be converted into rectangular displacements δx, δy.

The whole process is certainly very laborious; but when the result is attained it does furnish a great deal of information as to the character of the orbits adjacent to the orbit chosen for variation. I only carried the work through in one case, because I had gained enough information by this single instance. However, it does not seem worth while to record the numerical results in that case.

In the variation which has been described, C is maintained unchanged, but it is also possible to vary C. If C becomes $C+\delta C$, it will be found that the equations assume the form

$$\frac{d^2\delta q}{ds^2}+\Psi\delta q+\frac{\delta C}{V^2}\left(\frac{1}{R}+\frac{n}{V}\right)=0,$$

$$\frac{d}{ds}\left(\frac{\delta s}{V}\right)+\frac{2\delta q}{V^2}\left(\frac{1}{R}+\frac{n}{V}\right)+\frac{\delta C}{2V^2}=0.$$

But this kind of variation cannot be used with much advantage, for although it is possible to evaluate δq and δs for specific initial values of δC, δq, $d\delta q/ds$ at a specific initial point, only one single varied orbit is so deducible. In the previous case we may assign any arbitrary values, either positive or negative, to the constants denoted by a and b, and thus find a group of varied orbits.

§ 3. *A New Family of Periodic Orbits.*

In attempting to discover an example of an orbit of the kind suspected by Brown, I traced a number of orbits. Amongst these was that one which was varied as explained in § 2, although when the variation was effected I did not suspect it to be in reality periodic in a new way. It was clear that it could not be one of Brown's orbits, and I therefore put the work aside and made a fresh attempt, as explained in § 1. Finally, for my own satisfaction, I completed the circuit of this discarded orbit, and found to my surprise that it belonged to a new and unsuspected class of periodic. The orbit in question is that marked 33·5 in fig. 3, where only the half of it is drawn which lies on the positive side of SJ.

It will be convenient to use certain terms to indicate the various parts of the orbits under discussion, and these will now be explained. Periodic orbits have in reality neither beginning nor end; but, as it will be convenient to follow them in the direction traversed from an orthogonal crossing of the line of syzygies, I shall describe the first crossing as the "beginning" and the second orthogonal crossing of SJ as the "end." I shall call the large curve surrounding the apex of the Lagrangian equilateral triangle the "loop," and this is always described in the clockwise or negative direction. The portions of the orbit near J will be called the "circuit," or the "half-" or "quarter-circuit," as the case may be. The "half-circuits" about J are described counter-clockwise or positively, but where there is a complete "circuit" it is clockwise or negative. For example, in fig. 3 the orbit 33·5 "begins" with a positive quarter-circuit, passes on to a negative "loop," and "ends" in positive quarter-circuit. Since the initial and final quarter-circuits both cut SJ at right angles, the orbit is periodic, and would be completed by a similar curve on the negative side of SJ. In the completed orbit positively described half-circuits are interposed between negative loops described alternately on the positive and negative sides of SJ.

Having found this orbit almost by accident, it was desirable to find other orbits of this kind; but the work was too heavy to obtain as many as is desirable. There seems at present no way of proceeding except by conjecture, and bad luck attended the attempts to draw the curve when C is 33·25. The various curves are shown in fig. 2, from which this orbit may be constructed with substantial accuracy.

In fig. 2 the firm line of the external loop was computed backwards, starting at right angles to SJ from $x = \cdot 95$, $y = 0$, the point to which 480° is attached. After the completion of the loop, the

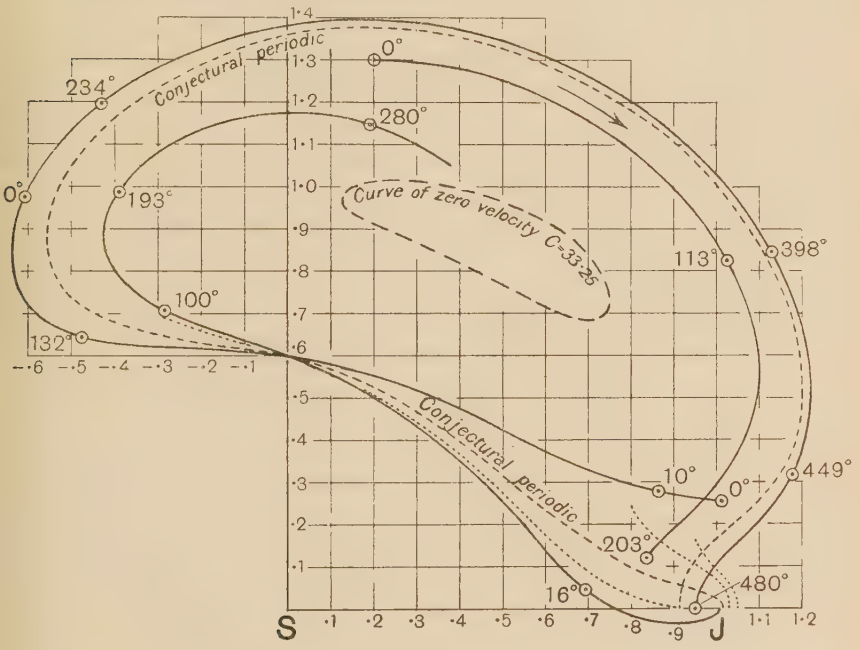


Fig. 2.—Orbits computed for the Case of $C = 33 \cdot 25$.

curve failed to come down close to J as was hoped, but came to the points marked 10° and 0°. The "beginnings" of two positively described quarter-circuits about J are shown as dotted lines, and an orbit of ejection, also dotted, is carried somewhat further. Then there is an orbit, shown in firm line, "beginning" with a negative half-circuit about J, and when this orbit had been traced half-way through its loop it appeared that the body was drawing too near to the curve of zero velocity, from which it would rebound, as one may say. This orbit is continued in a sense by a detached portion starting from a horizontal tangent at $x = \cdot 2$, $y = 1 \cdot 3$. It became clear ultimately that the horizontal tangent ought to have been chosen with a somewhat larger value

for y. From these attempts it may be concluded that the periodic orbit must resemble the broken line marked as conjectural, and as such it is transformed to fig. 3 and shown there as a dotted curve. I shall return hereafter to the explanation of the degrees written along these curves.

Much better fortune attended the construction of the orbit 33·75 shown in fig. 3, for, although the final perijove does not fall quite on the line of syzygies, yet the true periodic orbit can differ

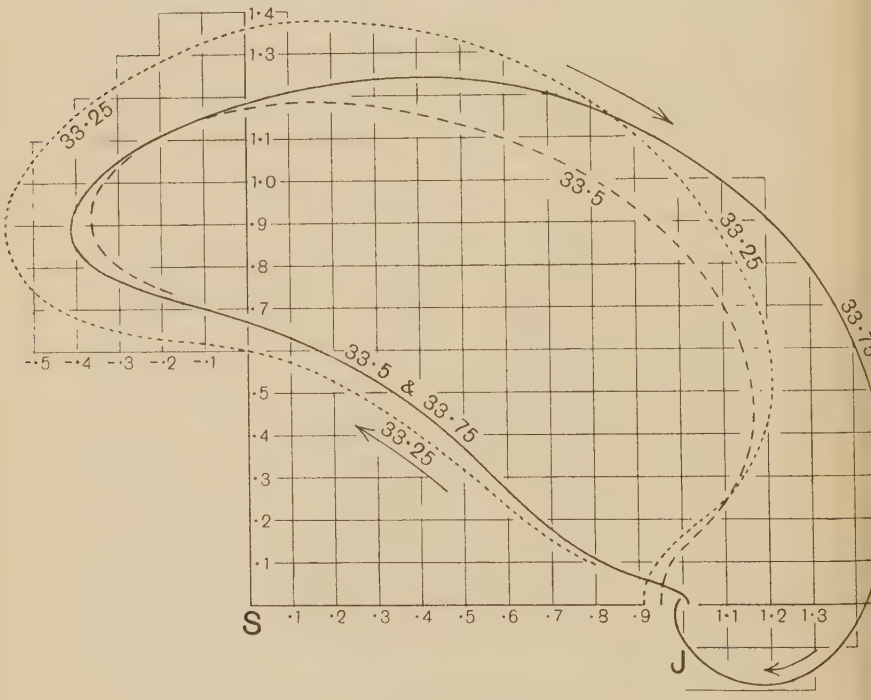


FIG. 3.—Three Periodic Orbits.

but little from that shown. It will be noticed that in this case the orbit "ends" with a negative half-circuit, and it is thus clear that if we were to watch the march of these orbits as C falls from 33·75 to 33·5 we should see the negative half-circuit shrink, pass through the ejectional stage, and emerge as a positive quarter-circuit when C is 33·5.

The three orbits shown in fig. 3 are the only members of this family that I have traced. It will be noticed that they do not exhibit that regular progress from member to member which might have been expected from the fact that the values of C are equidistant from one another. It might be suspected that they are

really members of different families presenting similar characteristics, but I do not think this furnishes the explanation.

In describing the loop throughout most of its course the body moves roughly parallel to the curve of zero velocity. For the values of C involved here that curve is half of the broken horse-shoe described in my paper on "Periodic Orbits" (*Scientific Papers*, vol. iv. p. 11, or *Acta Math.*, vol. xxi. (1897)). Now, for $\nu = 10$ the horse-shoe breaks when C has fallen to 34·91, and below that value each half of the broken horse-shoe, which delimits the forbidden space, shrinks. Now, since the orbits follow the contour of the horse-shoe, it might be supposed that the orbits would also shrink as C falls in magnitude. On the other hand, as C falls from 33·5 to 33·25, our figures show that the loop undoubtedly increases in size. This latter consideration would lead us to conjecture that the loop for 33·75 should be smaller than that for 33·5. Thus, looking at the matter from one point of view, we should expect the orbits to shrink, and from another to swell as C falls in value. It thus becomes intelligible that neither conjecture can be wholly correct, and we may thus find an explanation of the interlacing of the orbits as shown in my fig. 3.

It is certain from general considerations that families of orbits must originate in pairs, and we must therefore examine the origin of these orbits, and consider the fate of the other member of the pair.

It may be that for values of C greater than 33·75 the initial positive quarter-circuit about J is replaced by a negative half-circuit; but it is unnecessary for the present discussion to determine whether this is so or not, and it will suffice to assume that when C is greater than 33·75 the "beginning" is as shown in my figure. The "end" of 33·75 is a clearly marked negative half-circuit, and this shows that the family originates from a coalescent pair of orbits "ending" in such a negative half-circuit, with identical final orthogonal crossing of SJ in which the body passes from the negative to the positive side of SJ.

This coalescence must occur for some critical value of C between 34·91 and 33·75, and it is clear that as C falls below that critical value one of the "final" orthogonal intersections will move towards S and the other towards J.

In that one of the pair for which the intersection moves towards S the negative circuit increases in size; in the other in which it moves towards J the circuit diminishes in size, and these are clearly the orbits which have been traced. We next see that the negative circuit vanishes, the orbit becomes ejectional, and the motion about J both at "beginning" and "end" has become positive.

It may be suspected that when C falls below 33·25 the half-circuits round J increase in magnitude, and that the orbit tends to assume the form of a sort of asymmetrical double figure-of-8, something like the figure which Lord Kelvin drew as an illustration of his graphical method of curve-tracing.*

* *Popular Lectures*, vol. i., 2nd ed., p. 31; *Phil. Mag.*, vol. 34, 1892, p. 443.

In the neighbourhood of Jove the motion of the body is rapid, but the loops are described very slowly. The number of degrees written along the curves in fig. 2 represent the angles turned through by Jove about the Sun since the moment corresponding to the position marked 0°. Thus the firm line which lies externally throughout most of the loop terminates with 480°. Since this orbit cuts SJ orthogonally, it may be continued symmetrically on the negative side of SJ, and therefore while the body moves from the point 0° to a symmetrical one on the negative side Jove has turned through 960° round the Sun, that is to say, through $2\frac{2}{3}$ revolutions.

Again, in the case of the orbit beginning with a negative half-circuit, shown as a firm line, Jove has revolved through 280° by the time the point so marked is reached. We may regard this as continued in a sense by the detached portion of an orbit marked with 0°, 113°, 203°; and since 280° + 203° is equal to 483°, we again see that the period of the periodic orbit must be about 960°, or perhaps a little more.

In the cases of the other orbits more precise values may be assigned. For $C = 33{\cdot}5$, the angle nT (where T is the period) is 1115° or 3·1 revolutions of Jove; and for $C = 33{\cdot}75$, nT is 1235° or 3·4 revolutions.

It did not seem practicable to investigate the stability of these orbits, but we may suspect them to be unstable.

The numerical values for drawing the orbits $C = 33{\cdot}5$ and $33{\cdot}75$ are given in an appendix, but those for the various orbits from which the conjectural orbit $C = 33{\cdot}25$ is constructed are omitted. I estimate that it is as laborious to trace one of these orbits as to determine fully half a dozen of the simpler orbits shown in my earlier paper.

Although the present contribution to our knowledge is very imperfect, yet it may be hoped that it will furnish the mathematician with an intimation worth having as to the orbits towards which his researches must lead him.

The librating planets were first recognised as small oscillations about the triangular positions of Lagrange, and they have now received a very remarkable extension at the hands of Professor Brown. It appears to me that the family of orbits here investigated possesses an interesting relationship to these librating planets, for there must be orbits describing double, triple, and multiple loops in the intervals between successive half-circuits about Jove. Now, a body which describes its loop an infinite number of times, before it ceases to circulate round the triangular point, is in fact a librating planet. It may be conjectured that when the Sun's mass ν is yet smaller than 10, no such orbit as those traced is possible. When ν has increased to 10, probably only a single loop is possible: for a larger value a double loop may be described, and then successively more frequently described multiple loops will be reached. When ν has reached 24·9599 a loop described an infinite number of times must have become possible, since this is

the smallest value of ν which permits oscillation about the triangular point. If this idea is correct, and if N denotes the number expressing the multiplicity of the loop, then as ν increases $dN/d\nu$ must tend to infinity; and I do not see why this should not be the case.

These orbits throw some light on cosmogony, for we see how small planets with the same mean motion as Jove in the course of their vicissitudes tend to pass close to Jove, ultimately to be absorbed into its mass. We thus see something of the machinery whereby a large planet generates for itself a clear space in which to circulate about the Sun.

My attention was first drawn to periodic orbits by the desire to discover how a Laplacian ring could coalesce into a planet. With that object in view I tried to discover how a large planet would affect the motion of a small one moving in a circular orbit at the same mean distance. After various failures the investigation drifted towards the work of Hill and Poincaré, so that the original point of view was quite lost and it is not even mentioned in my paper on "Periodic Orbits." It is of interest, to me at least, to find that the original aspect of the problem has emerged again.

Appendix.

Numerical results of Quadratures.

$C = 33{\cdot}5$.

Perijove $x_0 = 1{\cdot}0171$, $y_0 = -{\cdot}0034$, taken as zero.
Time from perijove up to $s = -2{\cdot}1$ is given by $nt = 9^\circ\ 25'$.

s.	x.	y.	ϕ.	$2n/V$.
$-2{\cdot}1$	$+{\cdot}8282$	$+{\cdot}0980$	$+66^\circ\ 10'$	$2{\cdot}408$
$-2{\cdot}0$	·7409	·1467	55 53	·829
$-1{\cdot}9$	·6625	·2084	48 36	·876
8	·5894	766	46 3	·768
7	171	·3457	46 55	·655
6	·4425	·4124	49 46	·584
5	·3641	744	53 39	·568
4	·2814	·5306	57 56	·613
3	·1948	805	62 8	·728
2	049	·6243	65 51	·930
1	$+{\cdot}0126$	628	68 38	$3{\cdot}251$
$-1{\cdot}0$	$-{\cdot}810$	979	69 46	·760
$-{\cdot}9$	·1747	·7330	68 7	$4{\cdot}598$
·85	·2207	526	65 13	$5{\cdot}240$
·8	653	754	60 1	$6{\cdot}133$
·75	·3068	·8035	50 51	$7{\cdot}377$
$-{\cdot}725$	$-{\cdot}3252$	$+{\cdot}8203$	$+44\ 2$	$8{\cdot}139$

s.	x.	y.	ϕ.	$2n/V$.
			° ′	
− ·7	− ·3412	+ ·8395	+ 35 17	8·944
·675	537	611	24 33	9·664
·65	617	848	12 27	10·129
·625	644	·9096	+ 0 13	·224
·6	620	344	− 10 56	·009
·575	552	584	20 31	9·655
·55	448	811	28 30	·205
5	161	1·0220	40 48	8·448
·45	·2806	571	49 38	7·872
·4	405	869	56 51	·460
·3	·1518	1·1326	68 4	6·961
·2	− ·0565	626	76 47	·730
·1	+ 421	791	83 58	·647
·0	·1419	842	− 90 0	·633
+ ·05	919	830	π + 87 21	·630
·1	·2418	797	84 54	·626
·15	915	742	82 38	·609
·2	·3410	669	80 31	·572
·3	·4389	470	76 31	·432
·4	·5353	203	72 33	·201
·5	·6295	1·0869	68 16	5·912
·6	·7208	461	63 29	·605
·7	·8081	·9974	58 8	·313
·8	902	404	52 12	·055
·9	·9656	·8748	45 39	4·842
1·0	1·0326	006	38 22	·671
1	889	·7181	30 11	·540
2	1·1321	·6280	20 46	·435
3	585	·5318	9 38	·326
35	642	·4821	π + 3 16	·250
4	641	322	π − 3 40	·141
45	577	·3826	11 5	3·983
5	448	343	18 44	·758
55	257	·2881	26 8	·460
6	011	446	32 39	·100
65	1·0723	038	37 33	2·701
7	408	·1650	40 4	·291
+ 1·75	+ 1·0087	+ ·1267	π − 39 12	1·893

Time from $s = 1·75$ to perijove given by $nt = 5° 58'$.
Co-ordinates of perijove $x = ·9501$, $y = - ·0029$.

The following additional positions were calculated backwards from a perijove at $x = \cdot 95$, $y = 0$, $\phi = \pi$.

x.	y.	ϕ.
+ ·9500	+ ·0000	π + 0° 0′
512	531	π − 22 30
647	797	30 52
756	966	34 48
874	·1127	37 37
1·0128	436	40 37
390	738	40 56
649	·2043	39 12
893	360	35 51
1·1114	693	31 16
463	·3412	20 10
+1·1661	+ ·4186	π − 8 40

This supplementary orbit becomes indistinguishable in a figure of moderate size from the preceding orbit, which is therefore accepted as being periodic. The period is given by $nT = 1115^\circ \cdot 4 = 3 \cdot 1$ revolutions of Jove.

C = 33·75.

This orbit was computed from a conjectural starting-point which seemed likely to lead to the desired result; the computation was finally carried backwards from the starting-point. The co-ordinates of perijove were found to be $x_0 = 1 \cdot 0106$, $y_0 = \cdot 0006$, which may be taken as virtually on the line of syzygies. The motion from perijove is direct.

s.	x.	y.	ϕ.	$2\pi/V$.
perijove	+1·0106	+ ·0006	0° 0′	very nearly
− ·35	·9652	403	66 38	1·140
− ·3	184	578	71 6	·635
− ·25	·8713	744	69 27	2·075
− ·2	251	936	65 3	·447
− ·1	·7391	·1444	54 15	·882
0·	·6625	·2084	47 0	·946
·1	·5911	785	44 44	·850
·2	202	·3490	46 0	·749
·3	·4465	·4165	49 13	·686
·4	·3685	791	53 29	·675
·5	·2858	·5352	58 10	·723
·6	·1987	844	62 52	·838
·7	081	·6265	67 13	3·036
·8	+ ·0147	622	70 49	·348
·9	− ·0805	929	73 11	·834
1·0	·1764	·7213	73 25	4·631
1·1	− ·2713	·7525	69 17	6·090

s.	x.	y.	ϕ.	$2n/V$.
1·15	− ·3173	+ ·7721	63° 50′	7·333
1·2	601	977	53 25	9·236
1·225	791	·8140	45 6	10·360
1·25	951	332	33 54	11·840
1·275	·4064	553	19 53	12·955
1·3	118	796	+ 4 42	13·412
1·325	108	·9046	− 9 14	·174
1·35	043	287	20 35	12·599
1·375	·3936	513	29 25	11·945
1·4	800	723	36 21	·364
1·45	466	1·0096	46 23	10·471
1·5	082	416	53 25	9·849
1·6	·2227	940	62 21	·034
1·7	·1317	1·1356	67 59	8·347
1·8	− ·0377	696	72 2	7·618
2·0	+ ·1563	1·2184	79 17	6·140
2·2	·3547	407	−88 13	4·966
2·4	·5541	300	π+81 54	·182
2·6	·7487	1·1845	71 49	3·665
2·8	·9322	057	61 40	·305
3·0	1·0989	·9956	51 24	·052
3·2	1·2429	·8573	40 54	2·873
3·4	1·3588	·6946	29 55	·751
3·6	1·4402	·5123	18 1	·682
3·8	797	·3168	π+ 4 28	·670
4·0	674	·1181	π− 12 14	·733
4·1	377	+ ·0227	23 43	·806
4·2	1·3894	− ·0646	35 38	·910
4·3	208	·1366	52 23	3·027
4·35	1·2787	635	62 47	·068
4·4	322	817	74 47	·063
4·45	1·1829	892	π−88 15	2·983
4·5	332	845	+77 25	·780
4·55	1·0863	676	63 8	·477
4·6	448	399	49 32	·101
4·65	108	034	36 18	1·683
4·7	+ ·9867	− ·0598	21 1	1·234
perijove	·990	+ ·011	about 49	

The orbit is not rigorously periodic, but an extremely small change at the beginning would make it so. The period is given by $nT = 1234°·6 = 3·43$ revolutions of Jove.

Micrometrical Measures and Focal Peculiarities of Nova Lacertæ (Espin). By E. E. Barnard. (Plate 9.)

This star was discovered visually by Rev. T. E. Espin on 1910 December 30 at Tow Law, England. It was then between the 7th and 8th magnitudes.

According to *H.C.O. Bulletin*, No. 439, it was not visible on photographs taken 1910 November 17, but showed on those of November 23 and December 7 as a 5·00 magnitude star of the same brightness as 9 Lacertæ. A photograph taken 1887 December 1 which shows faint stars, according to the above *Bulletin* did not show the Nova. It is not stated what magnitudes were shown, and hence whether the star ought to have been on the plate.

In *A.N.* 4468 I have shown that the Nova was visible on photographs of mine as far back as 1893 October 11. Its position as measured on one of these photographs, 1907 August 7, agreed closely with that measured in the sky with the micrometer of the 40-inch, as will be seen by the following comparison:—

From the photograph of 1907

α 1911·0 = $22^h\ 32^m\ 11^s{\cdot}78$ δ 1911·0 = $+52^\circ\ 15'\ 19''{\cdot}7$

with the 40-inch $22^h\ 32^m\ 11^s{\cdot}79$ $+52^\circ\ 15'\ 19''{\cdot}8$

Both positions depend on that of the star A. G. Cambridge (Mass.) 7788.

Photographs made by Dr. Max Wolf on 1894 January 9 also show the Nova (*A.N.* 4466).

In the *Astrophysical Journal*, **33**, 410–417, 1911 June, Professor Frost has given a valuable account of the spectroscopic and other observations made here of the Nova in the early part of its appearance.

For the future identification of this star, and to lay a good foundation for the detection of proper motion hereafter, I have measured its position with respect to some of the faint stars near it. These measures were made with the micrometer of the 40-inch refractor. Bad weather interfered very much with the observations, which in some cases were made with difficulty. The results, however, should fix the place of the star very closely.

Following are the measures. Under good conditions there are some fainter stars in the region covered, but I have not thought it necessary to measure them:—

Micrometric Measures of the Positions of the Nova.
(*See Chart, p.* 672.)

		Nova and *p*.	
		°	″
1911·021 Jan.	8	287·18	171·78
·027	10	287·35	172·10
·079	29	287·34	171·89
·232 Mar.	26	287·48	172·28
1911·090		287·34	172·01

There is a fainter star 2″ or 3″ north of p and a 16 magnitude star 5″ ± following. Measures of the first of these are:—

p and small star north (13·7 magnitude).

			°	″
1911·462	June	18	351·00	2·80
·468		20	347·92	3·30
·495		30	350·92	2·98
·506	July	4	347·49	2·96
1911·480			349·33	3·01

Nova and o.

			°	″
1911·021	Jan.	8	304·61	147·15
·079		29	304·49	147·35
·084		31	304·75	147·19
1911·061			304·62	147·23

Nova and b.

			°	″
1911·021	Jan.	8	263·20	83·61
·027		10	262·95	83·86
·060		22	263·38	83·63
1911·036			263·18	83·70

Nova and a.

			°	″
1911·008	Jan.	3	323·59	121·62
·021		8	323·37	121·37
·027		10	323·23	121·87
·060		22	323·17	121·34
·079		29	323·18	121·15
1911·039			323·31	121·47
1911·487	June	27	323·14	121·61
·495		30	323·23	121·51
·500	July	2	323·28	121·61
·506		4	323·35	121·59
·539		16	323·33	121·63
1911·505			323·27	121·59

Nova and d.

			°	″
1911·021	Jan.	8	325·05	64·04
·027		10	324·13	63·79
·060		22	325·15	64·12
·079		29	325·57	64·09
1911·047			324·98	64·01

Nova and c.

			°	″
1911·021	Jan.	8	350·20	126·74
·060		22	350·17	126·41
·079		29	350·41	127·15
1911·053			350·26	126·77

Nova and m (15·5 magnitude).

			°	″
1911·021	Jan.	8	331·18	37·92
·468	June	20	331·30	39·15
·591	Aug.	4	331·26	39·11
·659		29	330·04	39·42
1911·435			330·94	38·90
1911·821	Oct.	27	329·10	...

Mag., June 20 and Aug. 29, 15·5; Oct. 27, 15 (clouds).

Nova and e.

			°	″
1911·021	Jan.	8	356·90	81·13
·027		10	357·09	81·21
·060		22	356·82	81·20
·079		29	356·91	81·42
1911·047			356·93	81·24

Nova and f.

			°	″
1911·021	Jan.	8	8·70	114·35
·060		22	8·59	114·37
·079		29	8·96	114·37
1911·053			8·75	114·36

Nova and *g*.

1911·021	Jan.	8	16°·96	110″·81
·060		22	16·92	110·81
·079		29	16·95	111·16
1911·053			16·94	110·93

Nova and *l*.

1911·021	Jan.	8	39°·24	52″·88
·060		22	39·47	52·63
·079		29	39·48	52·97
1911·053			39·40	52·83

Nova and *q*.

1911·084	Jan.	31	145°·08	[94″·92]
·290	Apr.	16	144·19	94·12
·309		23	144·88	93·53
·468	June	20	145·12	93·77
1911·263			144·82	93·81

First distance omitted from mean. Note says, "Faint in clouds."

Nova and *k*.

1911·021	Jan.	8	77°·47	62″·45
·027		10	78·17	62·61
·060		22	77·35	62·25
·079		29	77·56	62·64
1911·047			77·64	62·49

Nova and *r*.

1911·084	Jan.	31	97°·23	121″·54
·270	Apr.	9	96·93	121·35
·290		16	97·24	121·65
·309		23	97·09	121·27
1911·238			97·12	121·45

a and D.

1911·270	Apr.	9	252°·00	224″·62
·290		16	251·95	224·17
·309		23	251·91	224·26
·314		25	252·05	224·45
1911·296			251·98	224·38

D and *s*.

1911·309	Apr.	23	251°·28	18″·01
·314		25	251·44	18·25
·333	May	2	251·96	17·96
1911·319			251·56	18·07

a and *t*.

1911·391	May	23	274°·94	148″·91
·405		28	275·18	149·06
·424	June	4	275·14	149·25
1911·407			275·09	149·07

d and *m*.

1911·577	July	30	137°·35	25″·13
·591	Aug.	4	137·73	25·19
·659		29	134·86	25·09
1911·609			136·65	25·14
1911·667	Sept.	1	135·65	...
·821	Oct.	27	137·24	...
1911·744			136·44	...

Mag., July 30 and Aug. 29, 15·5; Oct. 27, 15 (clouds).

a and *v*.

1911·577	July	30	98°·55	29″·67
·591	Aug.	4	99·73	30·12
·615		13	100·38	30·15
·634		20	101·78	30·64
·659		29	98·76	29·94
1911·615			99·84	30·10
1911·667	Sept.	1	100·61	...

Mag., July 30, 15; Aug. 4, 14·7; Aug. 13, 20, 29, and Sept. 1, 15.

The measures of the Nova and a were repeated to see if there was any noticeable indication of parallax. The determination of the parallax of this star later by photography ($\pi = +0''.013$) by Professor Slocum (*Astrophysical Journal*, March 1912) shows that it would be hopelessly lost in the above observations.

The following measures were made on 1911 January 3. In each case there was one setting for angle and two settings for single distances:—

	°	″	m
a and f	79·90	91·59	13·5
a ,, d	140·10	57·70	13·7
a ,, c	62·60	57·97	13·5
a ,, g	85·60	105·23	13
a ,, l	119·10	120·29	13·5
a ,, k	122·30	157·91	13·5
a ,, e	104·10	70·64	13·3
a ,, b	185·20	107·73	13·7

It was from the above measures that the position of the Nova given in *A.N.* 4468 was determined on the plate of 1907 August 7.

It must be understood that these cannot (from the few measures) have the accuracy of the measures of the same stars with respect to the Nova. They must be considered as only approximate. They are given here as a matter of record.

Brightness of the Nova.

During the measures observations were made of the brightness and colour of the Nova.

Mr. Parkhurst has kindly supplied me with his visual photometric measures (with the 40-inch) of the brightness of the following small stars near the Nova, which are some of those measured by me. His results depend upon the assumption that star a, B.D. $+51^\circ$ 3420, is of the 8·80 magnitude.

a =	8·80	d =	12·20
D	10·09	r	12·93
p	11·57	q	12·46
b	12·07		

Partly with the aid of these magnitudes, I have determined the following values for the brightness of Nova Lacertæ.

1911.	C.S.T. h	m	m
Jan. 3	9	40	7·1
8	7	40	7·9
10	6	30	8·2
22	6	20	8·8
29	6	10	8·6
31	6	30	8·7
Feb. 28	8	10	9·0
Mar. 12	6	10	9·5
Apr. 9	15	50	9·6
16	16	00	9·9
23	16	00	10·0
May 2	15	30	10·0
7	15	35	10·2
16	15	20	10·3
23	15	00	10·6
28	14	50	10·5
June 4	15	00	10·5
18	14	45	10·6
20	14	20	10·5
27	14	50	10·6
30	15	00	10·9
July 2	15	25	10·8
4	15	10	10·9
11	15	00	10·6

1911.	C.S.T. h	m	m
July 16	12	30	10·8
21	15	00	10·8
23	15	00	10·7
30	13	20	11·1
Aug. 4	13	30	11·2
13	14	10	11·0
20	14	00	10·5
22	10	50	10·6
29	9	00	10·9
Sept. 1	11	15	10·4
3	8	50	10·7
19	11	30	10·7
Oct. 3	11	20	10·8
20	9	30	10·8
29	7	0	11·6
Nov. 8	5	15	11·4
11	5	30	11·2
17	5	30	11·2
24	4	20	11·0
25	4	20	11·2
Dec. 3	4	40	11·0
4	5	30	11·1
19	8	20	10·9

These observations seem to indicate that the light of the Nova fluctuated somewhat at times.

Estimated Magnitudes of the Comparison Stars.

Following are the means of the independent estimates of magnitude made January 3, January 8, January 29, and January 31, for each of the stars measured, except for *a* and D.

Star.	Mag.	Nights.	Star.	Mag.	Nights.
b	13·1	(4)	*m*	15·5	
c	13·2	(4)	*o*	13·1	(2)
d	13·1	(4)	*p*	12·4	(2)
e	13·0	(4)	*q*	14·6	(2)
f	13·3	(4)	*r*	13·5	(1)
g	13·0	(4)	*t*	$=b=13·1$	
k	13·1	(4)	*s*	12·4	
l	13·2	(4)	*v*	14·9	(5)

These are fainter than Mr. Parkhurst's values. This is partly due to the fact that the early estimates (perhaps due to the low altitude) were decidedly fainter than the later ones—the early observations being made in a hazy and broken sky. It is doubtful if they should have been included in the table.

With the aid of the plates with the 6-inch Willard lens of the Lick Observatory on 1893 October 11, and the 10-inch Bruce lens on 1907 August 7, 1909 August 22 (6-inch), and 1909 August 24, the following estimates of the magnitudes of the stars on the diagram have been obtained by the aid of Mr. Parkhurst's magnitudes:—

m		m
b = 11·9		k = 12·2
d = 11·9		l = 12·2
e = 12·1		o = 12·2
f = 12·9		p = 12·2
g = 12·2		

These do not seem to be satisfactory, and are about one magnitude brighter than my estimates in the sky.

Brightness of the Nova before the Outburst.

If we use these values and the visual estimates of magnitude, the following two sets of results would be obtained from the photographs for the brightness of the nova before the outburst. As will be seen, the two sets differ by about ½ magnitude:—

	m.	m.
1893 Oct. 11	12·7	13·2
1907 Aug. 7	12·9	13·6
1909 Aug. 22	12·8	13·3
1909 Aug. 24	12·7	13·3
	12·8	13·3

Perhaps it will be safest to adopt the mean of these, which is 13·1 magnitude. Neither set appears to verify my supposition (*A.N.* 4468) that the star fluctuated in its light previous to the outburst, though the comparisons with the individual stars seem to show this to be a fact.

Focal Peculiarities of the Nova.

Careful focal measures were also made when circumstances would permit.

I have already, in *A.N.* 4468, called attention to the fact that the star had two distinct and sharp foci, one of which gave a more or less colourless image at the regular focus of the telescope, while the other presented a beautiful and equally sharp crimson image

outside of the main focus. This second, crimson, image is due to the bright Hα line in the spectrum of the star In this peculiarity it resembled Nova Geminorum of 1903. In all probability the same feature is present in all Novæ at the proper stage of their decline if carefully looked for. As in the case of Nova Geminorum, this crimson image was of rather short duration and has entirely disappeared, not, I think, from the increasing faintness of the star, but from a regular physical change. Had the weather and the position of the Nova permitted, I had hoped to determine very closely the time of disappearance of this second image. All that I am able to say is that it disappeared between January 31 (perhaps February 28) and April 9; on the first date it was present, and on the second it had disappeared. On the latter date the star had become bluish-white. This colour was very strikingly noticed on April 16.

Mr. Parkhurst's photographs with the objective-prism show that by the middle of April the spectrum of the star had changed to the nebular condition, though the change was not very pronounced. By May 1 it was certain. Observations by Wright at the Lick Observatory show that the change had taken place even earlier. (See *L. O. Bulletin* 194.)

I had hoped to watch for this change, which would be indicated by a lengthening of the focus for the star, but between January 31 and May 16 the conditions were not suitable for exact focal measures from the bad weather and low position of the star. On May 16 the focus had noticeably lengthened, as will be seen from the following measures, which were made with a magnifying power of 700 diameters on the 40-inch telescope:—

1911 May 16. Focus of the Nova = 11·4 (6) mm.
„ „ „ Focus of the star a = 5·9 (5) „

\+ 5·5 mm.

The focus of the Nova, therefore, at this observation was 5·5 mm. greater than that for an ordinary star. This is in accord with the change to the nebular condition as indicated by Mr. Parkhurst's objective-prism photographs (*Astrophysical Journal*, 33, 417).

Had circumstances permitted, I think the change to the nebular condition would have been shown in its earliest stages by the focal measures.

Perhaps in the case of an extremely faint Nova this important physical change—which seems to be a characteristic feature of these stars—would be detected more readily by focal measures than by any other means.

Perhaps it would be well to explain here how these measures are made. On the tube of an eyepiece, magnifying 700 diameters, is engraved a millimetre scale. The Nova is carefully focused by slipping the eyepiece in or out and the scale read. This is usually repeated five times. A similar set of measures is then made on a star near and not differing greatly in magnitude from the Nova.

In speaking of the Hα image, it has occurred to me that some of the discordances in the descriptions of the colour of the Nova as seen with the 40-inch and with smaller telescopes may be explained by this crimson image. In general, with small instruments the star has been described as red, while with the 40-inch there has been but little colour to it except from the crimson glow from the out-of-focus scarlet or crimson image. This difference between its colour in the large telescope and in smaller ones is doubtless due to the fact that the great focal length of the 40-inch separates the portion of its spectrum due to the Hα line, and forms the second image at 10 mm. ± farther out. The shorter the focus of the telescope the nearer together these two images would be, until they would no longer be separate in small telescopes and the star would appear more or less red and relatively brighter.

During the observed brighter stages of the Nova, the star *a* was bluish or greenish-white. This colour seemed to be accentuated by contrast with the reddish glow about the Nova. In the later observations the new star has been bluish-white like *a*.

The following focal measures were made of the Nova and the star *a*. *A* is the scale reading for the normal image of the Nova, and *B* for its crimson image, *C* is the similar scale reading for the star *a*.

	Nova.				*Star a.*		
	A. Normal Image. Crimson Glow.		B. Crimson Image. Greenish Glow.		C. White Image. No Glow.		B – A.
1911.	mm.	No. of Obs.	mm.	No. of Obs.	mm.	No. of Obs.	mm.
Jan. 3	9·2	2	18·2	2	8·9	3	+ 9·0
8	8·6	3	16·2	3	7·8	3	+ 7·6
22	1·6	5	11·7	5	2·8	5	+ 10·1
29	4·1	5	13·5	5	5·2	5	+ 9·4
31	5·6	4	14·5	4	5·8	4	+ 8·9
							+ 9·0

The last column shows that the crimson image of the nova was 9·0 mm. outside of the normal image.

If we tabulate the differences C – A we have the following residuals:—

−0·3 mm.
−0·8
+1·2
+1·1
+0·2

Mean: +0·3 mm.

From these last results it is evident that the focus for the normal image of the Nova at this time was the same as that for *a*, for the above residuals (mean ± 0·3 mm.) are too small, under the conditions, to mean much in the difference of focus of the two

stars. Indeed it is remarkable, considering the great focal length of the 40-inch (19·354 metres), how close such measures can be made.

The following table of focal measures shows that the focus for the Nova finally became greater, corresponding to that for a nebula :—

1911.	Nova. mm.	*a*. mm.	Nova—*a*. mm.	
May 16	11·4 (6)	5·9 (5)	+5·5	
,, 23	7·2 (9)	4·8 (9)	+2·4	
July 4	9·3 (5)	5·6 (5)	+3·7	
,, 30	8·5 (3)	4·3 (3)	+4·2	
Aug. 4	11·3 (2)	5·0 (2)	+6·3	
Sept. 1	8·3 (3)	6·7 (3)	+1·6	Measures discordant from bad seeing.
,, 19	9·0 (5)	4·0 (5)	+5·0	
Oct. 20	11·7 (5)	5·4 (3)	+6·3	
,, 29	9·9 (5)	4·5 (5)	+5·4	
		Mean	+4·5	

All the focal measures were made with a power of 700 diameters.

The peculiar nature of the Novæ and the mystery of their outbursts make it important that every possible information about them should be obtained and printed. I have therefore made as complete a record of the present Nova as circumstances would permit. The descriptions of its physical appearance are contained in the following notes.

Notes on the Physical Appearance of the Nova.

The following are extracts from my notes of the appearance of the Nova in the large telescope. The times are C.S.T. $6^h\ 0^m$ slow of G.M.T. :—

1911 Jan. 3. *A* [see page 666] orange white surrounded by a crimson glow. *B* fine, well-defined crimson or scarlet image with greenish glow about it $5'' \pm$ in diameter. *C* white, with no glow.

Jan. 8. Nova slightly tinged with red and surrounded with reddish glow. Half-moon shining.

Jan. 10. It was tinged with pink, but seeing too poor to make much out of it. Could not tell if the crimson image was present. Bad seeing and haze. It was noted that in the 4-inch finder the Nova appeared relatively half a magnitude brighter than in the large telescope.

Jan. 22. The Nova was more red than formerly. It seemed to be a whitish image veiled with a pinkish glow. Crimson image well seen—like a ruby, with a greenish-grey glow about it. The star *a* has always been bluish-white by contrast.

Jan. 31. Crimson image still visible.

Feb. 28, 8^h 5^m (covered with clouds earlier in the night). Star very low. The image of the Nova a little red at the top, and as it got nearer the horizon, the red seemed to smear all over the image, and the star grew rapidly fainter compared with *a*.

March 12. Very low—seeing bad. The upper part of the image of the Nova tinged with red—atmospheric dispersion (?). This did not occur in the case of *a*. On this night, immediately after observing the Nova, I examined *R Leporis* (Hind's celebrated crimson star).

Focus for *R Leporis* = 4·2 (3) mm.
Focus for ordinary star near 3·4 (3) ,,

There was nothing peculiar about the out-of-focus image (in or out) of *R Leporis*. Out of focus it was only a red blur.

April 9, 15^h 50^m. Crimson image not present. The Nova was white. Seeing very bad, and images very large. 16^h 30^m. It looked almost bluish-white like *a*.

April 16. Bluish-white, like *a*. No trace of either the crimson image or the crimson glow.

April 23. $15^h\pm$. It was bluish-white. Nothing different in the out-of-focus image from that of *a*, except that it was of course fainter. Seeing good. Sky good.

May 2. $15^h\pm$. It was dull and bluish-white. Seeing very bad.

May 7. Seeing excessively bad. The Nova and D were very much alike, but the Nova was bluer. The image when best seen seemed to be softer than that of either *a* or D—it was more hazy.

May 16. $15^h\pm$. Broken hazy sky. The Nova was woolly and bluish-white. Could not get the focus as hard and sharp as D or *a*. Measures of the focus were made on this date which show a change in the focal distance. These are given elsewhere in this paper.

May 23. It was bluish-white and hazy, and seemed to be better defined than on the 16th—not so woolly.

May 28, 16^h 40^m. White.

June 4, 15^h 0^m. Pale white—not so well defined as *a*.

June 18, 14^h 40^m. At the focus for *a* the Nova appeared like a strong bluish-white nebulosity 5″·3 in diameter with a nucleus. The star *a* was free from such glow.

June 20, 16^h 10^m. With *a* in good focus the Nova was a hazy bluish mass 5″ in diameter, with a nucleus. It was not possible to get the object sharply focused.

June 27, 16^h 50^m. It was more blurred than the other stars.

June 30, 15^h 0^m. It appeared decidedly fainter than when last seen. It was pale and bluish-white, and did not seem to be so hazy. It was fairly well defined on this sky. The focus seemed to be about that of *a*.

July 2, 15^h 25^m. Bluish-white and woolly.

July 4. It was very closely hazy or nebulous. Could not get it free of the fuzziness. Bluish-white. One coming across it accidentally would be struck at once by its haziness.

July 11, $15^h\ 0^m$. Image very fuzzy.

July 16, $12^h\ 30^m$. Bluish-white and hazy.

July 21. $15^h\ 10^m$. I thought it was brighter than formerly. It was bluish-white and hazy.

July 23, $15^h\ 0^m$. Hazy and bluish-white.

July 30, $13^h\ 15^m$. It was decidedly fainter, and bluish-white, and very hazy, and the haze being 3″ or 4″ in diameter. It was still woolly when in best focus. It was badly out of focus when *a* was in focus.

Aug. 4, $14^h\ 20^m$. Apparently closely nebulous. Very hazy and out of focus. Bluish-white.

Aug. 13, $14^h\ 10^m$. Bluish-white. Apparently closely nebulous.

Aug. 20, $14^h\ 0^m$. Very hazy and bluish-white.

Aug. 22, $10^h\ 45^m$. Very hazy and bluish-white.

Aug. 29, $9^h\ 0^m$. Bluish-white and closely nebulous.

Sept. 1, $9^h\ 0^m$. It seemed to be very much brighter. Bluish-white and very blurred. Seeing bad, but it was far more blurred and hazy than the other stars. It was quite easily seen in the 4-inch finder. $13^h\ 40^m$. It looked like a small bluish-white nebula, apparently about $1\frac{1}{2}$″ in diameter.

Sept. 3, $8^h\ 50^m$. Hazy and bluish-white.

Sept. 19, $11^h\ 30^m$. Bluish-white. When in the best focus it was closely hazy with a small nucleus.

Oct. 3, $11^h\ 20^m$. Bluish-white and a little hazy—perhaps not as much as formerly, but nearly full moon shining. A rough test showed the focus to be greater than that for *a*. The brightness of the Nova has remained constant for some time.

Oct. 20, $9^h\ 30^m$. Very blue and very hazy—like a very small brightish nebula, 1″ or 2″ in diameter, with a small nucleus. It was perhaps no fainter than when last observed.

Oct. 27, $10^h\ 15^m$. Like a small bluish nebula 1″ or $1\frac{1}{2}$″ in diameter. The brightness seems unchanged.

Oct. 29, $7^h\ 0^m$. Apparently less bright. Bluish-white with dense nebulosity $1\frac{1}{2}$″ or 2″ in diameter.

Dec. 19, $10^h\ 20^m$. Perhaps brighter. Whitish in colour. Very low in west and definition too poor to tell anything of its focus.

The following observations were made of the nova with the 60-inch reflector at Mt. Wilson, California, with 100-foot focus and full aperture. The times are P.S.T. 8^h slow of G.M.T.:—

Nov. 8, $7^h\ 15^m$. P.S.T. The Nova was pale white and looked like a small nebula 1″ or 2″ diameter. The focus seemed to be exactly the same as that for the small stars near.

Nov. 9, $7^h\ 40^m$. Resembled a very small nebula.

Nov. 11, $7^h\ 30^m$. Like a very small nebula. No definite colour—white.

Nov. 25, $6^h\ 20^m$. It was whitish and more hazy than the small stars. Seeing poor.

I was delighted to have the opportunity, which was kindly

afforded me by Professor Hale, to examine the Nova with the great reflector on Mt. Wilson. To my eye there was no essential difference in its appearance from that with the 40-inch refractor. It looked like a very small nebula 1″ or 1½″ in diameter. There was no difference, however, between its focus and that of the small stars near it. This result is, of course, what would be expected from a reflecting telescope. An exposure of 30 minutes was made on 1911 November 19, with the 60-inch (100 feet focus), [Plate 9], with a Seed 27 plate, in the hope of showing the ill-defined appearance of the Nova, but the image was not different from that of the other small stars. It is possible that a shorter exposure (which would give smaller images) might have been better for this purpose On November 17 Mr. Adams examined the Nova with a small hand spectroscope and found that it gave a nebular spectrum.

In all the foregoing observations where the nova is spoken of as being 1″ or 2″ in diameter and nebulous, the remarks refer to the image when in the best focus.

Some Focal Experiments.

At the suggestion of Dr. S. A. Mitchell, I have made some focal experiments on a few stars, a list of which was supplied me by Dr. Mitchell. These experiments do not necessarily bear on the peculiarities of Nova Lacertæ, and are simply given here as a matter of interest in connection with the subject of out-of-focus images.

The stars were tested with the 40-inch refractor on the night of 1911 July 4, and gave the following approximate results. The air was unsteady and the observations not as satisfactory as they might otherwise have been.

α Herculis (*Secchi's Type III.*).

The central portion of the out-of-focus image was crimson and reduced to a small spot, perhaps as small as the image at the regular focus. This was roughly 8½ mm. outside the focus. If the seeing had been good this crimson image would perhaps have been perfectly stellar. When in focus the image of the large star was slightly yellow with a dense system of crimson rays extending for 3″ to 4″ from it. The seeing was too poor to decide as to the character of the image of the companion star.

γ Draconis (*Type II.*).

In focus. No crimson rays. Image and glow were yellow. The central part of the out-of-focus image rather strong crimson in colour but not definite, perhaps as small as the image at the regular focus. This crimson image was roughly the same distance outside the normal focus as that of α Herculis.

SOUTH.

p. *f.*

NOVA LACERTÆ, 1893 OCT. 11.

E. E. Barnard.—6-in. Willard Lens Lick Observatory.
Enlarged 6 diam.

SOUTH.

p. *f.*

NOVA LACERTÆ, 1911 NOV. 19.

E. E. Barnard.—60-in. Reflector, Mt. Wilson Observatory, California.
Enlarged 1·2 diam.

β Lyræ (Bright Line, Type V.).

Nothing peculiar was seen in the focal image of this star. It was surrounded by a broken yellowish-white halo. A very beautiful small blue image was found at a point 19·5 mm. outside of the normal focus. This blue image was as pure in colour and as striking as the crimson image of Nova Lacertæ. The focal and extra-focal images of γ Lyræ (Type I.) were an exact duplicate of those of β Lyræ, so that it is apparent that any peculiarity in the spectrum of β did not enter into the observations of that star.

γ Cygni (Type II.).

Out-of-focus image similar to that of P Cygni, but the blue image would come to a focus ½ inch nearer to the normal image than in the case of P Cygni. The out-of-focus image was in a large dense glow. The regular image was yellowish with no suggestion of crimson, simply a dense small yellow glow.

P Cygni (Bright Line, Type V.).

At 43 ± mm. outside the regular focus the central core reduced to a deep purplish fairly well-defined image, which was surrounded by a dense whitish-yellow glow 5″ to 6″ in diameter. This image was formed much further out than for the other stars, and the blue image was more of a purple colour. Nothing out of the ordinary was visible in connection with the regular focal image of this star—it was *then* slightly yellow. It seems that this object was once (1600) one of the so-called novæ.

In none of these stars did the focal image deviate from the regular focus of an ordinary star. In this respect they did not partake of the peculiarity shown by the Novæ and the nebulæ, of a slight lengthening of the regular focus.

There is nothing abnormal in these results. They are in full accord with the colour-curve of the 40-inch as determined by Professor Fox (*Ap. J.*, **27**, 237) combined with the spectral types of the stars.

The Plates.

Plate 9, fig. 1, 6 in. Willard lens. Lick Observatory, Mt. Hamilton, California, 1893 October 11. Exposure $6^h\ 21^m$, from $8^h\ 41^m$ to $15^h\ 2^m$ Pacific standard time. This clearly shows the Nova as a small star of the 13th or 14th magnitude. The enlargement is 6 diameters. The star images are not good, because the Nova is over 4° from the centre of the plate. There is no trace of nebulosity in this region on the original negative.

Plate 9, fig. 2, 60-inch reflector (100 feet focus) of the Solar Observatory of the Carnegie Institution, Mt. Wilson, California, 1911 November 19. Exposure $0^h\ 30^m$, from $6^h\ 34^m$ to $7^h\ 4^m$ Pacific standard time. Enlargement 1·2 times. The original

Chart of Nova Lacertæ region, from the Micrometer Measures.

negative shows stars of perhaps the 16th magnitude. The Nova is readily identified by the aid of the chart.

Criticism of Theories concerning the Novæ

There are several theories concerning the Novæ that have been accepted by some astronomers which have never seriously appealed to me.

The assumption that the outburst is due to a star colliding with a nebula is one of these. From what we know of the nebulæ in general they are very diffused masses of gaseous matter whose limits in many cases are uncertain because of their diffusion and faintness. I reject from this criticism the planetary nebulæ which are very definite in outline, but we must also reject these from the theory because of their relative smallness and scarcity. The enormous quantity of nebulous matter shown to exist in space—and we suppose this to be in the region of the fixed stars—makes it not only possible but probable that stars may sometimes run into it. The main point that seems unreasonable is that such an encounter with a nebula should produce the sudden outburst of light. Because of the diffused and attenuated condition of the outlying portions of the nebula it seems more reasonable that the increase of light by friction should be gradual and occupy years, instead of a few hours as actually occurs in the case of the Novæ, or, in other words, the star should slowly brighten just as a meteor (relatively speaking) does in our own atmosphere. Such an encounter with a nebula would be a matter of many years of disturbance in the light of the star, and in the case of a more or less central encounter with a large nebula, a matter of ages.

The theory that the Novæ are produced by the sudden rush of a star into a nebula is unreasonable and should be rejected. Photography does not show that the Novæ are in nebulous regions except in the case of Nova Andromedæ, and even here the spectroscope throws doubt upon the nebulosity. One would rather accept the theory of the collision of one star with another, but this theory does not appeal to me. It would seem more reasonable that the outburst of light is produced by some sudden change in the star's physical condition by forces inherent in the star itself, and that no outside influence enters into the problem.

One explanation of the apparently moving waves of nebulosity that were shown by photography to be receding from Nova Persei of 1901, and which seems to have been received with much favour, is that the star was in a large nebula when the outburst occurred, and that the apparent recession of this matter from the Nova was simply due to the progressive motion of the light successively illuminating structural portions of the nebula as the light receded from the star, and not a motion of the matter itself.

This explanation seems to have been generally accepted and the parallax of the star fixed, from this theory, at 0″·01. Not only this, but this more than questionable value has been freely used in

connection with other problems in astronomy. In reality it would seem that we know nothing of the actual distance of Nova Persei. I cannot see how anyone can still hold this theory after having examined the photographs made by Ritchey and Perrine with the reflectors of the Yerkes and Lick Observatories, for the individual details are clearly shown on the successive photographs to be moving outward bodily from the star. It is asking too much to claim that these are not identical structures which are shown in the various photographs.

If those who have not examined the evidence given by the photographs will turn to Plates XVIII. and XIX. in the *Astrophysical Journal*, vol. xiv., and carefully compare the two plates taken by Ritchey with the two-foot reflector of the Yerkes Observatory, they will, I think, readily verify the following facts. The sharp-pointed projection marked *a* in Plate XIX., fig. 2, 1901 November 13, has moved outward bodily when compared with fig. 2, Plate XVIII., 1901 October 20. Other details of the nebulosity also unquestionably show that there is actual outward motion of the nebulosity. This is specially noticeable in the case of the small tuft just free of the edge of the nebula at *b*, also in the case of the region of *f*, and at other points. Of course in the interval of twenty-four days some changes have necessarily taken place in some of the details of the nebulosity, but allowing for this, essentially all the details in the two drawings can be reconciled. This progressive motion of individual details is even better shown in Plates VI., VII., VIII., IX., X., of vol. xv. of the *Astrophysical Journal*, where Mr. Ritchey has given additional pictures of the star and the receding nebulous features. To those who do not have access to the *Astrophysical Journal*, the motion of *a* is well shown in the two photographs reproduced in "The Heavens and their Story" by Mr. and Mrs. Maunder, Plate LIX.

I am aware that this theory of the successive illumination of the nebulous matter by the outgoing light from the star was forced upon astronomers by the enormous velocity that had to be accounted for if the masses were in real motion. Though the light and nebula theory might conveniently explain the motion, it is directly antagonistic to the facts and should be rejected. What we have to deal with in this case is probably real motion caused by some force not yet known. Indeed, I think this, like some of the abnormal phenomena of the comets, reveals to us the effects of new forces (call them that if you like) as yet unknown to us, but which we must take into consideration as our knowledge of the universe advances.

In my paper in *A.N.* 4468 it should have been stated that the plates of 1909 August 22 and 24 were taken by Professor D. W. Morehouse while helping me with the Bruce telescope in the summer of 1909. This fact was overlooked until my paper was in print. I gladly make the correction here.

Note on Nova Geminorum 1912.

Since this paper was written, another Nova (Nova Geminorum No. 2, 1912 March 12, by Enebo) has appeared. Fortunately this last star was discovered very early in its career and has given an unusual opportunity to study the early history of these bodies. The following few remarks concerning it may not be out of place here.

On March 13, with the 5-inch guiding telescope of the Bruce photographic telescope, at $13^h\ 30^m$ G.M.T. the star was white and did not appear any different from the other stars near it.

March 16, $13^h\ 0^m$. With the 40-inch telescope, there was a narrow fringing of crimson about the image. The out-of-focus image was crimson, but not small or well defined at any focus (seeing poor). Settings were made on this image at the point where it appeared most intense. The result agreed with the regular position of the Hα image. The normal focal image was a bright golden yellow.

March 18. The Hα image was strong, but well defined. On March 22, with better seeing, this image was small, stellar, and well defined,—better defined, even, than the normal image. Since the first observations this image has grown more intense and beautiful.

I have so far carefully measured the position of this Hα image with the 40-inch and a power of 700 diameters on six nights. The results below show the agreement in its position for the two Novæ :—

Nova Lacertæ . .	Hα image	+9·0 mm. (5 nights)
Nova Geminorum No. 2	Hα image	+9·3 mm. (6 nights)
	Mean	+9·2 mm.

Independent of theoretical considerations, for my eye the position of the focus of the Hα image with the 40-inch telescope is 9·2 mm. farther from the object-glass than the normal image for an ordinary star.

A rather striking peculiarity developed in the frequent observations of this star with the 5-inch Bruce guiding telescope. After the first few observations, it was noticed that when the star was some little distance from the centre of the field the distorted image, produced by spherical aberration (the eyepiece gives a fairly flat field of about $\frac{1}{2}^\circ$), consisted of an intense scarlet point on the side towards the centre of the field, from which the rest of the image diffused towards the edge of the field. This was very striking and attracted the eye at once, as the other stars, of course, did not show this feature (that is, the scarlet image). It was thus easy in the rough settings of the telescope for position to identify the Nova instantly. This peculiarity (due to the Hα line in the star's spectrum) is still a strong characteristic of the Nova, which is now of the 7·3 magnitude. So striking is this feature that if

one were sweeping in this region with the 5-inch, he would recognize the Nova the moment it entered the field. It would appear, from this, that a search might successfully be made for Novæ with such a telescope visually, just as one searches for comets, and the observer would instantly and more certainly pick out a Nova from the other stars than he would the ordinary comet, devoid of a tail, from the many nebulæ he would encounter.

The above remarks, of course, hold only for the period in which the bright Hα line is strong in the spectrum of the star. If we take the history of the recent Novæ as a guide, this would be perhaps anywhere from the time the star is 3 or 4 days old (dating from the time of the increase in its light) until it is 3 or 4 months old. Before and after this period, when this line is not present, the Novæ could not be found by this method. Nova Geminorum No. 2 could not have been recognised until after March 13, nor Nova Lacertæ after February of 1911. Nova Persei, of 1901, for some years has not shown any peculiarity visually that would distinguish it from any other star.

In the 40-inch telescope a Nova would be recognised visually in two stages of its career; when the bright Hα line was present, and when the star had entered the nebular phase. In the first case, by the peculiarity of its image, both focal and extra-focal; and in the second case, by the increase of the regular focus and the ill-defined appearance of the star. From this latter fact there are doubtless hundreds, if not thousands, of ancient—perhaps we should say old—Novæ that could be recognised in the 40-inch telescope now, if we are to judge from the present appearance of Nova Cygni (1876), Nova Aurigæ (1891), and Nova Sagittarii (1898).

Yerkes Observatory:
1912 *March* 7.

Nova Cygni (1876): *Correction.*

In my paper in *M.N.* for April 1912, there is an error of $0''\cdot4$ in the mean of the declinations near the top of page 533.

For $+42^\circ\ 23'\ 24''\cdot5$ *read* $24''\cdot9$.

This also corrects the declination on the line below, for 1911·0

from $+42^\circ\ 26'\ 7''\cdot7$ *to* $8''\cdot1$

This leaves a discordance of only $0''\cdot7$ between the measures of 1901 and 1911 instead of $1''\cdot1$.

Yerkes Observatory:
1912 *June* 5.

The Constitution of the Solar Corona. II. By J. W. Nicholson, M.A., D.Sc.

It has been shown in the first paper that a large part of the coronal spectrum can be interpreted in terms of the vibrations of an elementary substance called protofluorine, to which a definite atomic constitution was assigned; and in an earlier paper that the main nebular lines can be interpreted similarly by the help of nebulium, a substance of an analogous nature. This interpretation led to the prediction of a new nebular line at $\lambda 4353$, which has been found subsequently, by Dr. W. H. Wright, on a plate of the Orion spectrum taken in 1908 at the Lick Observatory. The present paper takes up the further consideration of the spectrum of protofluorine, and connects the sets of wave-lengths corresponding to various degrees of electrical separation within the atom.

The constant of nature in terms of which these spectra can be expressed appears to be that of Planck in his recent quantum theory of energy.* It is evident that the model atoms with which we deal have many of the essential characteristics of Planck's "resonators." They possess a principal frequency $\omega/2\pi$, where ω is the angular velocity of the electrons within the atom, and we have seen, in the discussion of spectra, that this is ordinarily the frequency of a strong line in the spectrum of an atom. It is not desirable, in this paper, to give any detailed account of the principles of Planck's theory, but we shall avail ourselves of the main result. This states that the energy possessed by a resonator is of the form $nh\nu$, where ν is the frequency of the resonator, n is an integral number, and h is an universal constant of nature, whose value, derived by Planck from measurements on the radiation of a black body, is

$$h = 6{\cdot}548.10^{-27} \text{ erg seconds.}$$

In other words, interchanges of energy are not continuous, so that it is not possible to represent ultimate dynamics by sets of differential equations. The interchanges of energy are, moreover, regulated in magnitude by the frequencies of the systems concerned in a curious way.†

We proceed to examine to what extent this view is in accord with the various protofluorine atoms, neutral and positively charged. The systems with negative charges present some difficulties.

If a connection can be established between the systems on this basis—that is to say, if the ratio of atomic energy to principal frequency follows a regular law,—the investigation will serve the double purpose of confirming the suggested origin of the spectra of

* *Ann. der Phys.*, 1900, Bd. i. p. 69; 1901, Bd. iv. p. 553.

† A new mode of presenting the theory has been suggested by Sir Joseph Larmor in his Bakerian Lecture, *Proc. Roy. Soc.* A., vol. lxxxiii. p. 82. A critical discussion has been given recently by Poincaré, *Journal de Physique*, Jan. 1912.

astrophysics, and of giving to Planck's theory an atomic foundation: a foundation of the kind which is now generally believed to be necessary, giving a concrete picture of the possible nature of a resonator.

The energy of an atom.—As more than one writer has pointed out, the absolute energy of an atom cannot be known. We can only deal with changes of energy, or energy relative to that of an arbitrary standard configuration.

Let us examine the energy of a "simple" atomic system, consisting of a ring of n equidistant electrons rotating in a circle of radius a about a positive charge ve, where e is the electronic charge. For a single electron, the equation of energy, as in the theory of orbits, is for an attractive force ve^2/r^2 at distance r, of the type

$$\tfrac{1}{2}mv^2 = \mathrm{D} - \int \frac{ve^2}{r^2}\,dr = \mathrm{D} + \frac{ve^2}{a},$$

where m and v are the mass and velocity of an electron, and D is a constant. When n electrons are present together, the distances apart are the various values of the magnitudes of $2a \sin r\pi/n$, so that the potential energy of the atomic system, which would be $-nve^2/a$ in the absence of mutual action between negative charges, is changed to

$$-\frac{ne^2}{a}(\nu - \tfrac{1}{4}\mathrm{S}_n),$$

where

$$\mathrm{S}_n = \sum_{r=1}^{n-1} \operatorname{cosec} r\pi/n.$$

The equation of energy is

$$\tfrac{1}{2}mnv^2 = \frac{ne^2}{a}(\nu - \tfrac{1}{4}\mathrm{S}_n) + \mathrm{D},$$

where D is the energy in a standard configuration. In D is included any intrinsic energy which the electrons and nucleus may possess—such, for example, as may be concerned in their expansion, if Schott's view of the expanding electron is accepted as the cause of the fineness of spectral lines.

By the condition of steady motion,

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}\mathrm{S}_n) = mav^2,$$

and the kinetic energy of motion of the electron being $\frac{1}{2}mnv^2$, the potential energy of æthereal strain is of the form

$$\mathrm{D} - mna^2\omega^2,$$

where D is unknown. We shall examine the values of $mna^2\omega^2$, which will for brevity be referred to as "the energy."

Energy of the neutral atom of protofluorine. — Numerical determinations of the energy can have only a limited accuracy on account of the range of uncertainty in the experimental values

of e and e/m. But it is now believed that the most accurate values are

$$e = 4{\cdot}7.10^{-10} \qquad e/mc = 1{\cdot}77.10^{-7}$$

the first being in electrostatic, and the second in electro-magnetic units.*

In the first paper the principal vibration, $q = \omega$, for the proto-fluorine atom has a wave-length 3987·1 tenth-metres, or $3{\cdot}987.10^{-5}$ cms. Thus

$$\omega = 2\pi c/(3{\cdot}987.10^{-5});$$

moreover,

$$\omega^2 a^3 = 3{\cdot}6235e^2/m,$$

so that after calculation

$$\omega a = 1{\cdot}623.10^8,$$

and the measure of energy/frequency is

$$mna^2\omega^2.2\pi/\omega = 154{\cdot}94.10^{-27}.$$

This is of a similar order of magnitude to Planck's unit, and is nearly 25 of these units. It could be regarded as 25 units without any serious change in the experimental values of e, e/m. If e/m were unaltered, e would need an alteration from 4·7 to 4·85 only. We shall suppose, as is necessary for the complete theory of which this paper gives a slender outline, that the ratio of energy to frequency is exactly 25 of these units. The energy concerned is the potential æthereal energy reckoned in defect of a certain standard.

The smallness of the measure is in accord with the view that protofluorine is one of the simplest forms of matter—that is to say, one of the simplest receptacles of energy which can exist.

Angular momentum.—It is possible to hold another view of Planck's theory, which may be briefly pointed out. Since the variable part of the energy of an atomic system of the present form is proportional to $mna^2\omega$, the ratio of energy to frequency is proportional to $mna^2\omega^2$, or $mnav$, which is the total angular momentum of the electrons round the nucleus. If, therefore, the constant h of Planck has, as Sommerfeld has suggested, an atomic significance, it may mean that the angular momentum of an atom can only rise or fall by discrete amounts when electrons leave or return. It is readily seen that this view presents less difficulty to the mind than the more usual interpretation, which is believed to involve an atomic constitution of energy itself.

Energy/frequency measures of non-neutral atoms.—Consider next the atoms with positive charges. For protofluorine, which retains 4 and 3 electrons respectively, we have from the first paper,

No. of electrons = 4, Principal line = λ4231,

No. of electrons = 3, Principal line = λ4586.

* An excellent summary may be found in Kaye and Laby, *Physical and Chemical Tables*, 1911, pp. 99–100.

The case of two electrons is not included, as the identification given for it was unsatisfactory and only provisional.

The uncertainty of the values of e and e/m makes it convenient to adopt another unit of energy/frequency. From the relations

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}S_n),$$
$$\omega = 2\pi c/\lambda,$$

where λ is the principal wave-length, we deduce

$$mna^2\omega^2.2\pi/\omega = \frac{mn}{c}\left\{\frac{e^2}{m}.(\nu - \tfrac{1}{4}S_n)(2\pi c)\right\}^{\frac{2}{3}}\lambda^{\frac{1}{3}},$$

so that energy/frequency is proportional to

$$n(\nu - \tfrac{1}{4}S_n)^{\frac{2}{3}}\lambda^{\frac{1}{3}},$$

which may be taken as its measure, with λ in tenth-metres instead of centimetres. We shall denote the function by E. Thus, for neutral protofluorine, with the identification suggested in the first paper,

$$E = n(\nu - \tfrac{1}{4}S_n)^{\frac{2}{3}}\lambda^{\frac{1}{3}} = 187{\cdot}04,$$

which is 25 units if the unit is of magnitude 7·482.

For protofluorine with four electrons, $n = 4$, $\nu - \frac{1}{4}S_n = 4{\cdot}0429$, $\lambda = 4231$,

$$E = 164{\cdot}19.$$

For protofluorine with three electrons, $n = 3$, $\nu - \frac{1}{4}S_n = 4{\cdot}4226$, $\lambda = 4586$,

$$E = 134{\cdot}29.$$

Within one part in 1000, the three systems give respectively

$$25, \quad 22, \quad 18$$

of Planck's units.

These are the first three members of the harmonic sequence

$$25, \quad 22, \quad 18, \quad 13, \quad 7, \quad 0,$$

which would, if it continued valid, give no units to the positive nucleus alone, as would be expected if intrinsic energy were neglected.

The numbers of units *per electron* of (energy)/(frequency) would become, for the five systems in question,

$$5, \quad 5\tfrac{1}{2}, \quad 6, \quad 6\tfrac{1}{2}, \quad 7,$$

exhibiting a regular gradation, so that if the law were valid, the value of $mna^2\omega.2\pi$ for the system with n electrons and a principal frequency f would be

$$\tfrac{1}{2}(15 - n)nfh,$$

and it becomes possible to express the whole spectrum of proto-fluorine in the form of a series, or a set of series, depending on the natural numbers, though the series is not of the types of Balmer, Kayser and Runge, or Rydberg. These series, however, relate to elements whose structure is more complicated than a single ring system.

If the law continues to be valid in this way, there will be a coronal line for the system with two electrons, leading, within about 1 part in 1000, to a value of E of $6\frac{1}{2}$ units per electron. The line $\lambda 5073$ satisfies this necessity, for it would lead to

$$E = 2(4{\cdot}75)^{\frac{2}{3}}(5073)^{\frac{1}{3}} = 97{\cdot}107,$$

so that the number of units of magnitude 7·482 is

$$(97{\cdot}107)/(7{\cdot}482) = 12{\cdot}98,$$

and the number per electron is 6·495, which is $6\frac{1}{2}$ to less than 1 part in 1000.

An identification for this line was suggested in the first paper, but marked as very doubtful. The line is so weak that a companion line cannot be expected, and the agreement in the present case is so close that the origin of $\lambda 5073$ seems established.

This investigation supplies the first instance of the method by which lines can be identified when they have companions either outside the visible spectrum, or too weak to be recorded.

The companion lines.—A correction will now be introduced to certain of the wave-lengths calculated in the first paper, the need for which has been kindly brought to my notice by Dr. T. J. I'A., Bromwich. It had already appeared, as the result of more complete investigation, that the discrepancy between observed and calculated wave-length ratios could not be entirely explained by the neglect of the magnetic effects. But Dr. Bromwich's correction overcomes this difficulty completely. The transverse vibrations of class zero in the paper all have their wave-lengths too high. The mass M of the positive nucleus, although not entering as a rule into the periods of transverse vibrations, does so in the exceptional case $h = 0$, whose proper period equation is

$$mq^2 = ve^2(1 + nm/\mathrm{M})/a^3,$$

where the term in m/M represents the correction.* The mass of the electron is m.

In each of the above cases, including that of neutral nebulium, an application of this correction makes the coincidence between calculated and observed values even more exact. The value of M is, of course, unknown, but it is an essential point of the view that the atomic weight of this substance is comparable with that of hydrogen, that the value of m/M should be very small. In fact,

* Vide *Monthly Notices*, Nov. 1911, p. 52, equation (9).

it is generally believed that the electrons do not contribute greatly to the atomic weight.

If H is the value of M for a hydrogen atom, we know from experimental measures that $m/H = \cdot00054$ approximately. Let us calculate the atomic weight of protofluorine which would make the wave-lengths exact to the tenth of a tenth-metre. Only rough limits can be assigned, as the coronal lines are sometimes uncertain to the extent of a tenth-metre or more.

For three electrons, the calculated value was $\lambda = 4313$ for 4311·3. Thus

$$(1 + 3m/M)^{\frac{1}{2}} = 1\cdot7/4311,$$

or

$$m/M = \cdot00026,$$

giving an atomic weight 2·1 for protofluorine.

For four electrons, the calculated and observed wave-lengths are 3804·5, 3800·9, the latter being the mean of Dyson's three results, Campbell * giving 3800·4.

We derive $m/M = \cdot00047$, giving an atomic weight about unity. The case of five electrons gives, with Dyson's observation, a somewhat smaller value, Dyson having found the line only in 1901, when it was very strong, at λ3387·9, but Campbell giving λ3390·7. Campbell's value again leads to an atomic weight nearly 2. Thus there is undoubted evidence that the atomic weight is comparable with that of hydrogen.

The same method applied to nebulium gives an atomic weight 1·3. We must note that there is doubt about the line λ3454 being the *third* line of the neutral atom, for there is not a term in m/M to explain the discrepancy which was found. But the origin of the following lines now seems definitely established:—

Coronal Line.	Origin.	Intensity.
5073	P*f*, $+3e$	1
4586	P*f*, $+2e$	4
4311	P*f*, $+2e$	2
4231	P*f*, $+e$	5
3987·1	P*f*, neutral	3
3800·9	P*f*, $+e$	3
3387·9	P*f*, neutral	12

We proceed now, in the attempt to give an exhaustive treatment of the spectra of neutral and positively charged atoms of protofluorine, to give the theory of the vibrations in the plane of a single ring atom.

Vibrations in the plane of the atom.—The analysis necessary to a determination of these vibrations is similar to that employed by Sir J. J. Thomson for his atomic systems, but the results are very different on account of the change in the law of force.

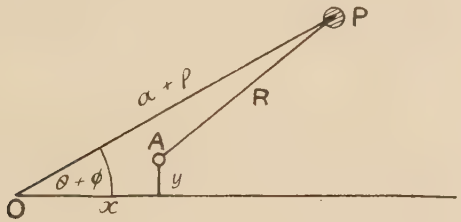
* *Lick Obs. Bull.*, No. 131, p. 11.

Moreover, we must suppose the positive nucleus to be capable of motion, for it is sometimes conspicuous in its effect on the periods. It is convenient, at the outset, to use the abbreviations in the following formulæ :—

$$\begin{aligned}
\mathrm{S} &= \Sigma_1^{n-1} \operatorname{cosec} s\pi/n, \\
8a^3\mathrm{A}'/e^2 &= 3\Sigma \operatorname{cosec} s\pi/n - \Sigma \operatorname{cosec}^3 s\pi/n, \\
8a^3\mathrm{A}_s/e^2 &= \operatorname{cosec} s\pi/n + \operatorname{cosec}^3 s\pi/n, \\
8a^3\mathrm{B}_s/e^2 &= \cos s\pi/n \operatorname{cosec}^2 s\pi/n, \\
4a^3\mathrm{C}/e^2 &= \Sigma \cos s\pi/n \operatorname{cosec}^2 s\pi/n \ (\cot s\pi/n + \tfrac{1}{2} \sin s\pi/n), \\
4a^3\mathrm{C}_s/e^2 &= \cos s\pi/n \operatorname{cosec}^2 s\pi/n \ (\cot s\pi/n + \tfrac{1}{2} \sin s\pi/n), \\
8a^3\mathrm{D}e^2 &= \Sigma \operatorname{cosec}^3 s\pi/n,
\end{aligned}$$

where all the summations range from unity to $n-1$.

Let a fixed origin be taken at the equilibrium position of the positive nucleus, whose co-ordinates related to any fixed axes in



the plane of the atom are (x, y) at time t. The *polar* co-ordinates of the rth electron may be denoted by $(a+\rho_r, \theta_r+\phi_r)$ where (ρ_r, ϕ_r) are small, and angles are measured from the axis of x. The radius of the undisturbed orbit is a. We first find the radial and transversal forces exerted by the moving nucleus on a typical electron in the position $(a+\rho, \theta+\phi)$, the nucleus being at (x, y). The arrangement is shown in the figure. The actual force is a radial attraction ve^2/AP^2 along P A, whose components along x and y are

$$x = -\frac{ve^2}{\mathrm{R}^3}\left\{(a+\rho)\cos(\theta+\phi) - x\right\}, \quad y = -\frac{ve^2}{\mathrm{R}^3}\left\{(a+\rho)\sin(\theta+\phi) \quad y\right\}$$

To the first order, the radial force along r increasing is

$$-ve^2\{a+\rho-x\cos\theta-y\sin\theta\}/\mathrm{R}^3,$$

and the transverse force increasing θ is

$$ve^2\{y\cos\theta-x\sin\theta\}/\mathrm{R}^3.$$

Writing, to the same order, $R = a + \rho - x\cos\theta - y\sin\theta$, the forces tending to increase r and θ become

$$\textit{Radial},\ -\nu e^2\{a - 2(\rho - x\cos\theta - y\sin\theta)\}/a^3,$$
$$\textit{Transversal},\ +\nu e^2\ \{y\cos\theta - x\sin\theta\}/a^3.$$

The forces R_p, Θ_p acting in the same directions on the pth electron by virtue of the presence of the others are, after Thomson, of the form

$$R_p = \frac{e^2 s}{4a^2} - \rho_p A' - \Sigma\rho_{p+s}A_s - a\Sigma\phi_{p+s}B_s,$$
$$\Theta_p = \Sigma\rho_{p+s}B_s - 2a\,C\,\phi_p + 2a\,\Sigma\phi_{p+s}C_s,$$

in terms of the quantities defined at the beginning of this section. The radial and transversal equations of motion of the pth electron are accordingly

$$m\left\{\ddot{\rho}_p - (a+\rho_p)(\omega+\dot{\phi}_p)^2\right\} = R_p - \frac{\nu e^2}{a^2} + \frac{2\nu e^2}{a^3}(\rho_p - x\cos\theta_p - y\sin\theta_p),$$
$$\frac{m}{a+\rho_p}\frac{d}{dt}\left\{(a+\rho_p)^2(\omega+\dot{\phi}_p)\right\} = \Theta_p + \frac{\nu e^2}{a^3}(y\cos\theta_p - x\sin\theta_p),$$

with the equation of steady motion

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}s_n).$$

If the oscillation has a period $2\pi/q$, so that all quantities relating to the variation from steady motion are proportional to e^{iqt}, then to the first order,

$$-mq^2\rho_p - 2ma\omega iq\phi_p = R_p - \frac{e^2 s}{4a^2} + \rho_p\left(m\omega^2 + \frac{2\nu^2}{a^3}\right) - \frac{2\nu e^2}{a^3}(x\cos\theta_p + y\sin\theta_p)$$
$$= -\rho_p A' - \Sigma\rho_{p+s}A_s - a\Sigma\phi_{p+s}B_s + \rho_p\left(m\omega^2 + \frac{2\nu e^2}{a^3}\right) - \frac{2\nu e^2}{a^3}(x\cos\theta_p + y\sin\theta_p)$$

and

$$2im\omega q\phi_p - maq^2\phi_p = \Sigma\rho_{p+s}B_s - aC\phi_p + a\Sigma\phi_{p+s}C_s + \frac{\nu e^2}{a^3}(y\cos\theta_p - x\sin\phi)_p.$$

If $A = A' - m\omega^2 - 2\nu e^2/a^3$, we get two sets of equations, whose first members are

$$(A - mq^2)\rho_1 + A_1\rho_2 + A_2\rho_3 + \ldots - 2ma\omega iq\phi_1 + aB_1\phi_2 + \ldots = -2\nu e^2(x\cos\theta_1 + y\sin\theta_1)/a^3 \quad (A)$$
$$2mi\omega q\rho_1 - B_1\rho_2 - B_2\rho_3 - \ldots + (C - mq^2)a\phi_1 - C_1 a\phi_2 - C_2 a\phi_3 \ldots = \nu e^2(y\cos\theta_1 - x\sin\theta_1)/a^3 \quad (B)$$

Consider now the motion of the positive nucleus. We find on

reduction that the forces (X_1, Y_1) exerted on it by any typical electron, tending to increase (x, y) are, to the first order,

$$X_1 = ve^2\{a\cos\theta + \rho\cos\theta - a\phi\sin\theta - x + 3\cos\theta\,(x\cos\theta + y\sin\theta - \rho)\}/a^3$$
$$Y_1 = ve^2\{a\sin\theta + \rho\sin\theta + a\phi\cos\theta - y + 3\sin\theta\,(x\cos\theta + y\sin\theta - \rho)\}/a^3$$

and these must be summed for all the electrons.

But
$$\Sigma_0^n \cos\theta_n = \Sigma_0^n \cos r\pi/n = 0,$$
$$\Sigma_0^n \sin\theta_n = \Sigma_0^n \sin r\pi/n = 0,$$

so that the first term of X_1 or Y_1 disappears on summation for all the electrons, and the total force is small—as it should be. Again,

$$\Sigma_0^n \cos\theta_n \sin\theta_n = 0,$$
$$\Sigma_0^n \cos^2\theta_n = \Sigma_0^n \sin^2\theta_n = \tfrac{1}{2}\Sigma_0^n(1 + \cos 2\theta_n) = \tfrac{1}{2}n,$$
$$\Sigma_0^n(3\cos^2\theta_n - 1) = \Sigma_0^n(3\sin^2\theta_n - 1) = \tfrac{1}{2}n,$$

so that the total forces on the positive nucleus, increasing x and y, are

$$ve^2\left\{ -2\Sigma\rho_n\cos\theta_n - a\Sigma\phi_n\sin\theta_n + \frac{nx}{2}\right\}/a^3,$$
$$ve^2\left\{ -2\Sigma\rho_n\sin\theta_n + a\Sigma\phi_n\cos\theta_n + \frac{ny}{2}\right\}/a^3,$$

and the equations of motion of the nucleus, of mass M, are

$$-Mq^2x = ve^2\left\{\frac{nx}{2} - 2\Sigma\rho_n\cos\theta_n - a\Sigma\phi_n\sin\theta_n\right\}/a^3$$
$$-Mq^2y = ve^2\left\{\frac{ny}{2} - 2\Sigma\rho_n\sin\theta_n + a\Sigma\phi_n\cos\theta_n\right\}/a^3$$

or, if
$$a = \tfrac{1}{2}n + Mq^2a^3/ve^2,$$

then
$$ax = 2\Sigma_1^n\rho_n\cos\theta_n + a\Sigma\phi_n\sin\theta_n,$$
$$ay = 2\Sigma\rho_n\sin\theta_n - a\Sigma\phi_n\cos\theta_n,$$

and (x, y) are of order m/M in comparison with any ρ or ϕ. They will be zero of necessity in all symmetrical vibrations, such, for example, as those of class zero. This is seen at once, because the ρ's are all equal, and also the ϕ's, in this class of vibration, and come outside the summations. We seek to solve these equations, with (A) and (B), in the old manner. Let ω be any nth root of unity, and write

$$\rho_2 = \rho_1\omega, \quad \rho_3 = \rho_2\omega, \ldots .$$
$$\phi_2 = \phi_1\omega, \quad \phi_3 = \phi_2\omega, \ldots .$$

Then
$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2,$$
$$ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1,$$

where

$$\sigma_1 = \Sigma_1^n \omega^{r-1} \cos\theta_r, \quad \sigma_2 = \Sigma_1^n \omega^{r-1} \sin\theta_r, \quad \theta_r - \theta_{r-1} = 2\pi/n.$$

We must now determine the values of σ_1 and σ_2. We find

$$\sigma_1 \pm i\sigma_2 = \Sigma_1^n \omega^{r-1} e^{\pm i\theta_r},$$

and if k be integral or zero, defining the "class" of the vibration as in the case of transverse vibrations, $\omega = e^{2ik\pi/n}$. Thus

$$\sigma_1 \pm i\sigma_2 = e^{\pm i\theta_1} \Sigma_{1n} e^{2ir\pi(k\pm 1)/n}.$$

If $k = \mp 1$, this summation has the value n, but is otherwise zero when k is not greater than n.

Thus σ_1 and σ_2 separately vanish unless $k \pm 1$ is zero, and therefore x and y vanish, so that the mass of the positive nucleus does not affect the periods. Vibrations of classes ± 1 are therefore exceptional. If $k = 1$, then

$$\sigma_1 + i\sigma_2 = 0, \quad \sigma_1 - i\sigma_2 = ne^{-i\theta_1},$$

or

$$\sigma_1 = \tfrac{1}{2}ne^{-i\theta_1}, \quad \sigma_2 = \tfrac{1}{2}ine^{-i\theta_1}.$$

If $k = -1$, then

$$\sigma_1 - i\sigma_2 = 0, \quad \sigma_1 + i\sigma_2 = ne^{i\theta_1},$$

or

$$\sigma_1 = \tfrac{1}{2}ne^{i\theta_1}, \quad \sigma_2 = -\tfrac{1}{2}ine^{i\theta_1}.$$

There is no loss of generality involved in taking $\theta_1 = 0$. Also

$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2$$
$$ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1.$$

Vibrations of classes other than ± 1.—In these vibrations, x and y are zero, and the solution is comparatively simple. The typical equations (A) and (B) become, with ρ_p for ρ_1 and so on,

$$(A - mq^2)\rho_p + A_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + aB_1\phi_{p+1} + \ldots = 0$$
$$2im\omega q\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2)a\phi_p - C_1 a\phi_{p+1} - \ldots = 0$$

These equations are of the same form as those obtained by Sir J. J. Thomson for his atomic model, but the coefficients are different. The ratios assumed for the ρ's and ϕ's are the same, and therefore after his analysis we may write down the period equation for the vibrations of class k as

$$(A - mq^2 + L_k)(C - mq^2 - N_k) = (M_k - 2m\omega q)^2,$$

where ω is the angular velocity of rotation, and

$$8a^3 L_k = e^2 \Sigma_1^{n-1} \cos 2rk\pi/n (\operatorname{cosec} r\pi/n + \operatorname{cosec}^3 r\pi/n),$$
$$8a^3 M_k = e^2 \Sigma \sin 2rk\pi/n \cos r\pi/n \operatorname{cosec}^2 r\pi/n,$$
$$8a^3 N_k = 2e^2 \Sigma \cos 2rk\pi/n \cos r\pi/n \operatorname{cosec}^2 r\pi/n (\cot r\pi/n + \tfrac{1}{2}\tan r\pi/n).$$

Moreover, $C = N_0$. The function A is different from that of Thomson, and has the value

$$A = A' - m\omega^2 2ve^2/a^3,$$

where A', defined at the beginning of the investigation, is readily reduced to the form

$$A' = -L_0 + e^2 S_n/2a^3,$$

and, finally,

$$A = -(L_0 + 3m\omega^2)$$

by the use of the equation of steady motion.

The period equation therefore becomes

$$(L_k - L_0 - 3m\omega^2 - mq^2)(N_0 - N_k - mq^2) = (M_k - 2m\omega q)^2$$
$$k = 0,\ 2,\ 3, \ldots .$$

and the functions occurring in it have been given for numbers of electrons up to 6 by Thomson. The equation differs from his by containing the term $3m\omega^2$. Nagaoka has given an investigation of this system with a stationary nucleus, but errors have crept in, and the resulting period equation is of the wrong type.

For vibrations of class $k = 0$, the equation reduces to

$$q^2(q^2 + 3\omega^2) = 4\omega^2 q^2,$$

giving $q = 0$, $q = \pm\omega$.

Thus the period equation of the simplest and most important vibration in the plane is merely $q = \omega$, as we have hitherto stated.

Even for vibrations of classes ± 1, this equation must give a fair approximation to the periods, if the mass of the nucleus is really the source of nearly the whole mass of the atom, as the present theory of material constitution supposes. But we shall find that it is not a sufficient equation. Especially in the case of nebulium, some of these vibrations of class 1 are important spectral lines.

Vibrations of Classes ± 1.—The equations in this more complex case are again of two types:—

$$(A - mq^2)\rho_p + A_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + aB_1\phi_{p+1} + \ldots$$
$$= -2ve^2(x\cos\theta_p + y\sin\theta_p)/a^3,$$
$$2im\omega q\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2)a\phi_p - C_1 a\phi_{p+1} - \ldots$$
$$= ve^2(y\cos\theta_p - x\sin\theta_p)/a^3,$$

where

$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2, \quad ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1,$$
$$(\sigma_1,\ \sigma_2) = \tfrac{1}{2}n(1, -i)e^{-i\theta_1} \qquad (k = -1)$$
$$-\tfrac{1}{2}n(1,\ i)e^{-i\theta_1} \qquad (k = 1)$$
$$\rho_2 = \omega\rho_1,\ \rho_3 = \omega\rho_2 \ldots, \quad \phi_2 = \omega\phi_1,\ \phi_3 = \omega\phi_2,$$

the ω in the last formulæ being the nth root of unity. Consider the case $k=1$. Then on reduction

$$x\cos\theta_p + y\sin\theta_p = \frac{n}{2a}(2\rho_1 + \iota a\phi_1)e^{\iota(\theta_p-\theta_1)}.$$

But $$\theta_p - \theta_1 = 2\pi(p-1)/n, \qquad \omega = e^{2\pi i/n},$$
$$e^{i(\theta_p-\theta_1)} = \omega^{p-1},$$

so that

$$x\cos\theta_p + y\sin\theta_p = \frac{n}{2a}(2\rho_p + ia\phi_p)$$

on reduction. In a similar manner,

$$y\cos\theta_p - x\sin\theta_p = \frac{n}{2a}(2i\rho_p - a\phi_p).$$

If therefore

$$\beta = \frac{n}{2a}\cdot\frac{ve^2}{a^3} = \frac{nve^2}{2a^3}\Big/\left(\frac{\mathrm{M}q^2a^3}{ve^2} + \tfrac{1}{2}n\right),$$

and if we include also the case $k=-1$, whose detailed working follows a similar analysis, we find the period equations to be derived from

$$(\mathrm{A}-mq^2)\rho_p + \mathrm{A}_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + a\mathrm{B},\ \phi_{p+1} + \ldots = -2\beta(2\rho_p \pm ia\phi_p).$$
$$2im\omega q\rho_p - \mathrm{B}_1\rho_{p+1} - \ldots + (\mathrm{C}-mq^2)a\phi_p - \mathrm{C}_1a\phi_{p+1} - \ldots = \beta(\pm 2\iota\rho_p - a\phi_p).$$

The upper sign refers to the case $k=1$, and the lower to $k=-1$. Thus

$$(\mathrm{A}-mq^2+4\beta)\rho_p + \mathrm{A}_1\rho_{p+1} + \ldots + (-2ima\omega q \pm 2ia\beta)\phi_p + a\mathrm{B}_1\phi_{p+1} + \ldots = 0$$
$$(2im\omega q \mp 2\iota\beta)\rho_p - \mathrm{B}_1\rho_{p+1} - \ldots + (\mathrm{C}-mq^2+\beta)a\phi_p - \mathrm{C}_1a\phi_{p+1} - \ldots = 0$$

These are again of the old form. The resulting period equation, derived as before, is

$$(\mathrm{A}-mq^2+4\beta+\mathrm{L}_k)(\mathrm{C}-mq^2+\beta-\mathrm{N}_k) = (\mathrm{M}_k - 2m\omega q \pm 2\beta)^2,$$

being different for the classes $+1$ and -1. This may be written as

$$(\mathrm{L}_k - \mathrm{L}_0 - 3m\omega^2 - mq^2 + 4\beta)(\mathrm{N}_0 - \mathrm{N}_k - mq^2 + \beta) = (\mathrm{M}_k - 2m\omega q \pm 2\beta)^2,$$

where the term in β^2 disappears on expansion.

Effectively, as m/M is small, we may write $\beta = m\omega^2\gamma$, where

$$\gamma = \tfrac{1}{2}n\nu^2(\nu - \tfrac{1}{4}S_n)^{-2}\left(\frac{\omega^2}{q^2}\right)\left(\frac{m}{M}\right),$$

and the equation becomes

$$\left(\frac{L_k - L_0}{m\omega^2} - x^2 - 3\right)\left(\frac{N_0 - N_k}{m\omega^2} - x^2\right) - \left(\frac{M_k}{m\omega^2} - 2x\right)^2 = \gamma\left\{\frac{L_0 - L_k - 4N_0 + 4N_k \pm 4M_k}{m\omega^2} + (3 \mp 5x)(1 \pm x)\right\},$$

where x is q/ω. The solution may be derived by continued approximation. If x_0 is a root of the equation

$$f(x) = 0$$

or

$$\left(\frac{L_k - L_0}{m\omega^2} - x^2 - 3\right)\left(\frac{N_0 - N_k}{m\omega^2} - x^2\right) - \left(\frac{M_k}{m\omega^2} - 2x\right)^2 = 0,$$

the more accurate solutions for classes ± 1 are

$$q/\omega = x_0 + \{\gamma/f^1(x_0)\}\{(m\omega^2)^{-1}(L_0 - L_k - 4N_0 + 4N_k \pm 4M_k) + (3 \mp 5x)(1 \mp x)\}$$

These are the values of q relatively to the moving ring of electrons.

Numerical calculations for the neutral atom.—In proceeding to calculation, we must remember, from the investigations of Schott, that no spectral line of a class higher than 2 will be found ordinarily, although those of class 3 may appear occasionally in strong non-neutral systems.

When $n = 5$, the following results may be obtained, if δ denotes $e^2/8a^3$.

$$\begin{array}{lll} M_0 = 0, & M_1 = 4{\cdot}856\delta, & M_2 = 2{\cdot}103\delta \\ L_0 = 17{\cdot}6788\delta, & L_1 = 0{\cdot}511\delta, & L_2 = -9{\cdot}352\delta \\ N_0 = 18{\cdot}8408\delta, & N_1 = 2{\cdot}9716\delta, & N_2 = -12{\cdot}395\delta \end{array}$$

Consider first the vibrations of class 2. Since by the equation of steady motion,

$$\delta/m\omega^2 = (28{\cdot}99)^{-1},$$

the period equation is

$$\left(x^2 + 3 + \frac{27{\cdot}031}{28{\cdot}99}\right)\left(x^2 - \frac{31{\cdot}236}{28{\cdot}99}\right) = \left(2x - \frac{2{\cdot}103}{28{\cdot}99}\right)^2$$

or

$$x^4 - 1{\cdot}1452x^2 + 0{\cdot}2902x - 4{\cdot}2428 = 0.$$

This equation, in common with *all* the quartics to which we are subsequently led, has one real and positive, one real and negative, and two complex roots. Thus there are always unstable

vibrations in the plane of the atom, and these vibrations supply the mechanism by which it is possible for the various positively and negatively charged systems to exist in presence of each other. When an unstable vibration has resulted in the ejection of one or more electrons from a system, the new system can again take them up until the unstable modes arise again. Thus in a vibrating gas of this type, there would ultimately be a kind of equilibrium established in the interchange of electrons, and spectral lines might appear in correspondence with all possible positive or negative degrees of charge.

The real roots of the quartic in the present case are

$$x = 1{\cdot}61, \ -1{\cdot}66,$$

or

$$q/\omega = 1{\cdot}61, \ -1{\cdot}66.$$

Since they are of class 2, the corresponding vibrations for an observer are derived from

$$q/\omega = 3{\cdot}61, \ 0{\cdot}39, \ 3{\cdot}66, \ 0{\cdot}34,$$

and since the line $q = \omega$ is at $\lambda 3987$, none of these vibrations are in the visible spectrum.

The real part of the complex roots, moreover, is half the sum of the real roots, or $-{\cdot}025$, and gives vibrations of about half the wave-length of the principal line, which are not in the spectrum. Thus no vibration of class 2 for this system could be perceptible.

Since the period equations for classes k, $n-k$ are identical, we may show that no vibration of class 3 could fall in the visible spectrum.

In general we shall find, in accordance with a remark in the first paper, that the number of vibrations in the plane which give lines falling in the visible spectrum, is very small. Except for the case $q = \omega$ of class zero, the only remaining possibilities for the neutral atom are the vibrations of classes ± 1.

If $q/\omega = x$, the quartic defining these vibrations is found to be

$$x^4 - 0{\cdot}95519x^2 + 0{\cdot}6696x - 1{\cdot}9945 = 0,$$

when the term containing γ is ignored. The two real roots are

$$x_0 = 1{\cdot}2827, \ -1{\cdot}5085.$$

As the existence of the term in γ cannot affect the first decimal place, the possible vibrations for an observer are given by

$$q/\omega = \pm 1 + 1{\cdot}3, \ \pm 1 - 1{\cdot}5,$$

and none are in the visible spectrum. We are reduced therefore to a consideration of the unstable vibrations. If they are of the type $x = \theta \pm i\phi$ relatively to the rotating electrons,

$$2\theta + 1{\cdot}2827 - 1{\cdot}5085 = 0,$$

or $\theta = \cdot 1129$, so that the vibrations for an observer, uncorrected for γ, have wave-lengths determinable from $q/\omega = 1\cdot 1129, \cdot 8871$.

Since the wave-length corresponding to $q = \omega$ is λ 3987, these approximate wave-lengths become 3582, 4493. Nothing in the spectrum is close to the second line, and the nearest wave-length to the first is Campbell's line $\lambda 3602$, which is much too strong, although not found by Dyson, to be due to a vibration of this character.

A somewhat laborious application of the correction for m/M indicates that the true wave-length is $3987/(1\cdot 1129 - 4\cdot 6\ m/\mathrm{M})$, and only an increase of about 8 tenth-metres is produced, so that we do not get Campbell's line.

We conclude, therefore, as might be expected from the large argument of the real exponential of these waves, that unstable vibrations of this type are not strong and continuous enough to give observable lines. But the formal verification given here seems necessary for completeness. We must notice that two vibrations for which q/ω is small are introduced when the positive mass occurs in the period equation; but these in the present case, as in most others, are of the unstable type, and manifest themselves with a frequency $q = \omega$, so that they coincide in wave-length with the principal line.

Protofluorine with two electrons.—If $\delta = e^2/8a^3$, we have for this system,

$$\mathrm{M}_0 = \mathrm{M}_1 = 0, \quad \mathrm{L}_0 = 2\delta, \quad \mathrm{L}_1 = -2\delta, \quad \mathrm{N}_0 = -\mathrm{N}_1 = \delta,$$

and the first approximation to vibrations of class 1 is given by

$$\left(x^2 + \frac{59}{19}\right)\left(x^2 - \frac{1}{19}\right) = 4x^2,$$

a soluble biquadratic. But the real roots are nearly equal to ± 1, and the imaginary roots have no real part. The correction to the latter is, in its real part, of magnitude $7\cdot 4\ m/\mathrm{M}$. The real roots lead to no vibrations in the visible range, and the corrected imaginary roots to no visible lines. If it might be supposed that the considerable radiation in this case, when an electron is departing, should be manifested, it must be remembered that the system is very weak, and that even one of the vibrations of class zero is missing, so that unstable vibrations can hardly appear, especially when absent from the neutral atomic spectrum.

Protofluorine with three electrons.—For this special case, since the quartics for classes k, $n - k$ are always identical except for a possible term in m/M, the quartic for class 2 is the first approximation to that for class 1, namely

$$x^4 - 0\cdot 9347x^2 + 0\cdot 1304x - 0\cdot 5276 = 0.$$

The roots are ultimately found to be $x = 1\cdot 1148, -1\cdot 2096$, so that for an observer, there are no vibrations of class 1 in the visible range of spectrum. For class 2 there are theoretically

two, but it is not likely that they can ever be observed, for the system is weak in the corona, and vibrations of class 2 are only important even to any moderate extent for negatively charged systems. Their wave-lengths are 5180, 5802.

Protofluorine with four electrons.—This is a much stronger system, whose principal line is λ4231. The quartic for class 2 is

$$x^4 - x^2 - 1{\cdot}8494 = 0,$$

whose real solution is $x = 1{\cdot}3960$. The vibration nearest to the visible spectrum is given to an observer as $q/\omega = 0{\cdot}6040$, and has a wave-length 7005, far in the infra-red.

The vibrations of class 3 accurately, and of class 1 approximately, satisfy

$$x^4 - 0{\cdot}9382x^2 + 0\ 3498x - 1{\cdot}1053 = 0,$$

which has real solutions $x = 1{\cdot}1926$, $-1{\cdot}3451$. With a principal line λ4231, we find that neither class 1 nor class 3 give any lines in the visible range.

Summary.—In the present paper, the suggested theory of the coronal spectrum has been put upon a definite basis which is in accord with the recent theories of emission of energy by bodies. It is indicated that the key to the physical side of these theories lies in the fact that an expulsion or retention of an electron by any atom probably involves a discontinuous change in the angular momentum of the atom, which is dependent on the number of electrons already present.

This view leads to a determination of spectral lines which is free from the dependence on the method of ratios.

It has been shown that coincidences between observed and calculated wave-lengths can be rendered exact by a process which shows at the same time that the atomic weights of nebulium and protofluorine are comparable with that of hydrogen.

The complete spectrum of protofluorine due to neutral or positively charged atoms is given in the table.

Wave-length.	Intensity.	Origin.
5073	1	P*f*, $+3e$
4586	4	P*f*, $+2e$
4311	2	P*f*, $+2e$
4231	5	P*f*, $+e$
3987·1	3	P*f*, neutral
3800·9	3	P*f*, $+e$
3387·9	12	P*f*, neutral

The origin of these lines is now fairly certain. All other lines of protofluorine in the spectrum must be due to negatively charged atoms.

Trinity College, Cambridge:
1912 *April* 28.

On the New Nebular Line at λ4353. By J. W. Nicholson, M.A., D.Sc.

At the meeting of the Society of 1912 March the writer announced the discovery of the new nebular line at λ4353 which had been predicted in his paper on "The Spectrum of Nebulium." * A plate of the spectrum of the Orion nebula, on which the line was found, and which had been taken with a long exposure at the Lick Observatory in 1908 by Dr. W. H. Wright, was also exhibited. In the meantime the line has been recorded again by Dr. Max Wolf, of Heidelberg, who has, in a letter, given an account of its discovery, and this brief note gives a record of some of the details of the observation.

The plate on which the line is shown was exposed at Heidelberg between 1912 January 20 and February 28, with an exposure of $40^h\ 48^m$. The most northern star of the Trapezium is in the centre of the photographed region, and the new line is visible fairly strongly, especially in the spectrum of the star and on both sides.

The wave-length in the Orion nebula, obtained by plotting from an iron curve, is 4353·9, which is, of course, too large, as all the lines in this nebula are shifted to greater wave-lengths, on account of the motion of the nebula.† But the correction is not so large as a tenth-metre.

The wave-length of the line on the Lick plate, as measured at the Cambridge Observatory by Mr. Stratton, is 4353·3, the value calculated in the paper being 4352·9.

Trinity College, Cambridge:
1912 *May* 8.

Photographic Magnitudes of Stars brighter than $9^m{\cdot}0$ *within* 5° *of the North Pole.* By F. W. Dyson and C. Davidson.

The determination of the photographic magnitudes of the stars in the Greenwich section of the Astrographic Catalogue is being made by comparison with Prof. Pickering's North polar sequence. As there are no stars brighter than $8^m{\cdot}9$ within 1° of the North Pole, the photographs taken with the Astrographic telescope have been supplemented by a series taken with the Franklin-Adams 6-inch lens, which well covers a field of 3° radius. Each field is photographed when its centre is at the same altitude as the pole, and the polar area containing Pickering's standards is photographed on the same plate with an equal exposure.

It was found to be desirable to amplify the number of stars of the polar area available for purposes of comparison so as to include

* *Monthly Notices*, vol. lxxii., No. 1 (November 1911).
† Keeler, *Lick Observatory Publications*, iii., 1894; Hartmann, *Ast. Jour.*, xv. 1902, p. 287.

all the stars brighter than $9^{m}\cdot0$. In the first instance this was attained by taking four photographs each with three sets of exposures, the telescope being set at the pole and at 3° on either side. The four plates were set at right ascensions 0^{h}, 3^{h}, 6^{h}, and 9^{h} respectively, so that each star was brought into from 8 to 12 different positions relatively to the axis of the lens. The magnitudes of about 100 stars within four degrees of the pole were thus determined. Using these stars as standards, the programme of comparing other fields with the polar area was proceeded with. This work necessitated the measurement of the standards many times, and it became possible to improve the determination of these standards. Altogether, results obtained from 25 plates were combined with the results obtained from the four original plates. In order that the stars should not always fall in the same part of the field, the exposures on the polar area were made with the telescope pointing 1° from the pole and at various right ascensions.

The magnitudes have been determined from the measured diameters of the images, such exposures being given that the images of $9^{m}\cdot0$ stars were in all cases black. The formula connecting magnitude and diameter has been taken as $m = C - k\sqrt{d}$, where d is the measured diameter of the image. This formula generally satisfies the measures very well over a range of at least three magnitudes. The constants of the formula for each plate depend essentially on Pickering's standards, on the first four plates these alone being used and on the subsequent plates those of the secondary sequence determined from them in addition.

From a comparison of the determinations on different plates, the probable error of a result from one plate has been found to be $\pm^{m}\cdot073$. The results from $6^{m}\cdot0$ to $9^{m}\cdot0$ may be accepted with confidence as in good accord with Pickering's standards as given in *Harvard Circular*, No. 170. A large difference is shown for the red star B.D. 87°, 51. The separate results of the Greenwich photographs are—

m.	m.	m.	m.	m.
6·46	6·71	6·44	6·46	6·38
6·43	6·46	6·61	6·59	6·58
6·51	6·56	6·89	6·30	6·55
6·59	6·50	6·34	6·45	
6·30	6·47	6·62	6·55	

A comparison of the magnitudes with those given in the "Aktinometrie der Sterne der B.D. bis zur Grösse 7·5 in der Zone 0° bis +20° Deklination," published by Prof. Schwarzschild in the *Abhandlungen der Königl. Gesell. der Wissenschaft zu Göttingen*, Neue Folge, Band viii., No. 4, shows a systematic difference of $+0^{m}\cdot10$ between the magnitudes $6^{m}\cdot0$ and $8^{m}\cdot0$. There is no indication that the differences vary with the colour of the star.

It should be stated that Ilford "Empress" plates were used, and that the colour correction for the lens appears to be nearly perfect from H_β to K. The lens is designed to cover a field of radius $7\frac{1}{2}°$, but was only used over a radius slightly above 3°, and was stopped down to half the aperture, to secure uniformity of focus and definition over the field.

Photographic Magnitudes of Stars near the North Pole.

No.	B.D. No.	R.A. 1900.	Dec. 1900.	Photog. Mag.	No. of Determinations.	H. C. 170. Seq. No.	H. C. 170. Mag.	H. – Gr.	Photog. Mag. Göttingen.	Gött. – Gr.
		h m s	° ′	m			m	m		
1	88° 8	1 22 33	+88 46	2·65	24	1s.	2·71			
2	86 269	18 4 33	86 37	4·42	25	1	4·47	+·05	4·34	−·08
3	85 383	22 21 19	85 36	5·20	15	2	5·24	+·04	5·14	−·06
4	85 19	0 55 2	85 43	5·49	6				5·55	+·06
5	86 344	23 27 49	86 45	5·68	35	3	5·74	+·06	5·70	+·02
6	86 272	18 7 47	87 0	5·82	31	4	5·91	+·09	5·92	+·10
7	86 51	3 33 55	86 20	5·95	20				6·12	+·17
8	88 4	0 55 38	88 29	6·42	34	5	6·39	−·03	6·44	+·02
9	85 78	5 9 52	85 35	6·44	6				6·51	+·07
10	88 71	12 14 23	88 15	6·45	33	2s.	6·38	−·07	6·39	−·06
11	87 107	12 13 56	86 59	6·47	29	3s.	6·54	+·07	6·59	+·12
12	87 51	6 53 44	87 12	6·51	23	1r.	6·69	+·18	6·66	+·15
13	85 409	23 54 46	86 9	6·55	19					
14	86 176	11 59 42	86 8	6·60	11				6·69	+·09
15	85 399	23 24 23	85 52	6·62	14				6·87	+·15
16	85 74	4 56 19	85 50	6·62	6				6·70	+·08
17	85 269	15 57 23	85 35	6·76	6				6·95	+·19
18	85 63	4 5 6	85 17	6·82	4				6·96	+·14
19	85 403	23 30 23	85 38	6·95	15					
20	89 13	7 58 2	88 56	6·99	31	6	6·98	−·01	7·14	+·15
21	86 17	0 59 6	86 37	7·00	31				7·25	+·25
22	88 64	11 4 14	88 11	7·18	33	7	7·18	·00	7·28	+·10
23	86 319	21 19 35	86 37	7·20	31				7·29	+·09
24	86 182	12 34 36	86 17	7·21	10				7·24	+·03
25	87 205	22 24 14	87 35	7·24	33				7·37	+·13
26	86 161	11 2 30	86 11	7·31	8				7·31	·00
27	85 401	23 27 30	86 0	7·32	16					
28	86 201	13 59 24	86 14	7·33	9				7·67	+·34
29	86 170	11 28 20	86 10	7·37	11				7·47	+·10
30	85 222	13 18 39	85 17	7·42	4				7·61	+·19

Photographic Magnitudes of Stars near the North Pole—continued.

No.	B.D. No.	R.A. 1900.	Dec. 1900.	Photog. Mag.	No. of Determinations.	H. C. 170. Seq. No.	H. C. 170. Mag.	H. – Gr.	Photog. Mag. Göttingen.	Gött. – Gr.
	°	h m s	° ′	m			m	m		
31	85 263	15 42 32	+85 9	7·42	5				7·79	+·37
32	85 80	5 29 54	85 9	7·43	4				7·54	+·11
33	85 384	22 21 41	85 43	7·47	15				7·61	+·14
34	85 45	2 23 22	85 22	7·50	5					
35	86 79	6 8 3	86 46	7·51	18				7·59	+·08
36	85 294	18 7 10	85 41	7·52	14				7·71	+·19
37	85 128	8 25 20	85 24	7·56	5					
38	85 41	1 58 48	85 16	7·56	6					
39	86 66	4 46 17	86 10	7·62	9					
40	88 112	19 22 30	88 59	7·69	34	2r.	7·73	+·04	7·79	+·10
41	86 217	14 49 38	86 22	7·69	19				7·99	+·30
42	86 347	23 57 19	86 29	7·71	22					
43	86 39	2 32 13	86 37	7·77	29					
44	86 193	13 32 25	85 47	7·82	3				8·10	+·28
45	86 335	22 42 19	86 46	7·84	35					
46	86 187	12 59 43	86 25	7·85	10				7·83	−·02
47	86 282	18 47 44	86 35	7·87	23				7·92	+·05
48	86 221	15 6 19	85 54	7·88	12					
49	85 161	10 20 45	84 55	7·90	4				7·80	−·10
50	87 101	11 54 37	87 33	7·97	32					
51	87 15	1 49 43	88 0	7·98	33					
52	86 25	1 40 19	86 26	8·00	27					
53	86 113	7 52 26	85 59	8·01	7				7·87	−·14
54	87 143	15 9 21	87 37	8·05	33				8·01	−·04
55	85 183	11 24 23	85 15	8·02	4				7·85	−·17
56	85 400	23 26 17	85 27	8·06	14					
57	85 249	15 9 9	85 31	8·10	7					
58	88 9	2 14 14	88 42	8·10	33	8	8·10	·00		
59	86 103	7 11 29	86 35	8·10	13					
60	86 143	9 58 12	86 19	8·15	8					
61	85 11	0 29 15	85 25	8·17	6					
62	85 359	21 6 30	85 29	8·17	11					
63	86 324	21 46 26	86 33	8·18	28					
64	88 104	17 53 52	88 15	8·18	33					
65	87 78	9 17 58	87 18	8·21	24					

Photographic Magnitudes of Stars near the North Pole—continued.

No.	B.D. No.	R.A. 1900.	Dec. 1900.	Photog. Mag.	No. of Determinations.	H. C. 170. Seq. No.	H. C. 170. Mag.	H. – Gr.	Photog. Mag. Göttingen.	Gött. – Gr.
	°	h m s	° ′	m				m	m	
66	87 180	19 14 31	+87 10	8·22	32					
67	87 169	18 4 23	87 25	8·23	31					
68	87 147	15 27 13	87 23	8·25	32					
69	85 9	0 26 12	85 46	8·28	17					
70	85 50	2 40 58	85 28	8·29	7					
71	86 159	10 57 0	86 5	8·31	7					
72	87 181	19 15 45	87 41	8·33	33					
73	86 264	17 32 35	86 57	8·34	27					
74	86 154	10 40 40	85 54	8·38	4					
75	87 122	13 25 8	87 5	8·40	33					
76	87 201	21 59 14	87 19	8·41	35					
77	86 152	10 18 33	86 34	8·42	9					
78	85 406	23 50 52	85 21	8·42	12					
79	86 9	0 37 8	86 24	8·43	20					
80	86 244	16 34 50	86 26	8·46	12					
81	86 275	18 26 10	86 32	8·46	22					
82	86 318	21 16 22	87 8	8·48	36					
83	87 83	9 43 57	87 3	8·49	23					
84	86 102	0 45 27	86 57	8·50	13					
85	85 412	0 2 14	86 14	8·52	18					
86	87 33	4 4 51	88 2	8·53	32					
87	88 77	13 26 44	88 4	8·55	33					
88	85 2	0 8 46	86 10	8·58	19					
89	88 86	14 18 5	87 52	8·59	34					
90	85 191	11 44 57	85 33	8·59	6					
91	85 367	21 50 53	85 59	8·61	18					
92	86 172	11 40 9	86 5	8·61	9					
93	87 99	11 47 27	86 47	8·61	15					
94	85 278	16 49 33	85 49	8·62	8					
95	88 76	13 4 30	88 11	8·65	33	3r.	8·62	– ·03		
96	85 337	20 3 2	85 36	8·65	10					
97	86 65	4 38 1	86 43	8·65	31					
98	86 54	3 52 36	86 40	8·67	29					
99	85 376	22 2 3	85 23	8 67	8					
100	88 2	0 16 5	88 53	8·68	3					

Photographic Magnitudes of Stars near the North Pole—continued.

No.	B.D. No.	R.A. 1900.	Dec. 1900.	Photog. Mag.	No. of Determinations.	H. C. 170. Seq. No.	H. C. 170. Mag.	H. – Gr.	Photog. Mag. Göttingen.	Gött. – Gr.
	°	h m s	° ′	m			m	m		
101	88 131	22 12 6	+88 58	8·68	33					
102	86 21	1 10 50	86 25	8·69	21					
103	88 13	2 42 15	88 34	8·69	33	9	8·70	+·01		
104	88 11	2 17 24	88 15	8·69	30					
105	85 266	15 49 50	85 33	8·69	5					
106	87 68	8 26 9	87 15	8·70	24					
107	87 217	23 42 52	87 47	8·71	33					
108	87 115	12 58 7	87 12	8·71	33					
109	86 177	12 6 53	86 16	8·71	11					
110	88 117	19 59 1	88 50	8·75	33					
111	85 248	14 55 51	85 42	8·75	7					
112	87 9	0 59 36	87 44	8·76	32					
113	87 12	1 18 4	88 3	8·77	33					
114	85 329	19 33 54	85 53	8·77	15					
115	88 105	17 51 33	88 44	8·80	33					
116	87 104	12 8 8	87 29	8·81	29					
117	85 320	19 0 34	85 29	8·83	12					
118	87 1	0 7 26	87 51	8·83	33					
119	88 60	10 19 26	88 23	8·83	31					
120	88 80	13 53 37	87 48	8·83	33					
121	87 26	2 58 32	87 33	8·84	32					
122	85 53	3 4 32	86 3	8·86	16					
123	89 3	3 18 54	89 41	8·87	37	10	8·89	+·02		
124	87 187	20 25 8	87 38	8·88	33					
125	86 96	6 50 11	86 29	8·88	14					
126	88 115	19 54 3	88 34	8·89	33					
127	87 16	1 56 8	88 12	8·91	32					
128	86 256	17 12 5	86 13	8·92	9					
129	87 46	6 35 29	87 32	8·92	27					
130	87 85	10 3 59	87 47	8·92	29					
131	86 7	0 27 6	87 15	8·93	31					
132	86 38	2 25 15	86 33	8·93	20					
133	88 114	19 43 20	88 41	8·94	34	4r.	8·97	+·03		
134	86 126	8 47 1	85 57	8·94	6					
135	86 211	14 30 2	86 3	8·94	5					

Photographic Magnitudes of Stars near the North Pole—continued.

No.	B.D. No.	R.A. 1900.	Dec. 1900.	Photog. Mag.	No. of Determinations.	H. C. 170. Seq. No.	H. C. 170. Mag.	H – Gr.	Photog. Mag. Göttingen.	Gött. – Gr.
	°	h m s	° ′	m			m	m		
136	85 347	20 27 10	+85 57	8·95	8					
137	86 86	6 23 1	86 3	8·96	4					
138	85 354	20 50 5	85 18	8·96	5					
139	86 199	13 54 39	86 1	8·96	8					
140	86 67	4 50 51	86 44	8·97	21					
141	88 110	18 48 33	88 38	8·97	33					
142	85 239	14 22 1	85 1	8·98	6					
143	85 340	20 13 37	85 28	8·99	6					
144	86 171	11 38 27	85 54	8·99	5					
145	86 277	18 40 40	86 9	9·00	13					
146	85 390	22 43 18	86 8	9·00	12					
147	88 39	7 16 37	87 57	9·00	12					
148	86 332	22 22 1	87 5	9·02	9					
149	86 14	0 49 12	86 47	9·05	31					
150	86 43	2 50 14	87 8	9·05	8					
151	86 242	16 20 9	86 3	9·05	5					
152	85 330	19 35 43	85 53	9·07	14					
153	87 151	16 5 29	87 45	9·10	10					
154	86 44	2 49 54	87 1	9·11	6					
155	86 222	15 9 9	86 17	9·20	8					
156	89 21	12 42 9	89 14	9·28	28					
157	89 18	11 34 57	89 29	9·32	31	11	9·42	+·10		

Royal Observatory, Greenwich:
1912 *June* 14.

A proposal for the Comparison of the Stellar Magnitude Scales of the different Observatories taking part in the Astrographic Catalogue. Third Note. The Algiers Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The previous notes are in *Mon. Not.*, lxix. p. 392, and *Mon. Not.*, lxxii. p. 464. The object of the present note is firstly to illustrate further the use of the method for detecting systematic errors of the magnitude scale; secondly, to confirm and extend the result found on pp. 476–7 of this volume from Stratonoff's zones and the Algiers maps, viz., that there is a relative increase of faint stars in the direction of the "In-Vertex" ($94^\circ + 12^\circ$), and a decrease in that of the "Out-Vertex" ($274^\circ - 12^\circ$). The counts are, as before, due to the kind volunteer aid of Miss Riddle.

2. In Table I. are given the number of images for each whole magnitude of the Algiers scale in Volume VI. of their publications, centres at $+1^\circ$ (the volume for plates with centres actually on the equator not having yet been received). The counts were made for each plate separately, and the totals checked with those for the whole plate: but for the present purpose it is sufficient to tabulate each hour of R.A.

TABLE I.

Number of Images recorded under each whole magnitude in the Catalogue of Algiers $+1^\circ$. (Tome VI.)

Mag. / R.A.	<7·0.	8·0.	9·0.	10·0.	11·0.	12·0.
0^h	8	7	104	96	269	290
1	7	3	109	72	301	249
2	8	3	93	92	271	372
3	10	21	133	105	473	405
4	16	28	177	172	341	575
5	52	57	305	254	654	709
6	50	71	539	499	1343	2891
7	40	66	818	584	1975	3368
8	27	35	356	359	1051	1163
9	11	38	141	130	422	530
10	14	42	155	103	381	255
11	22	58	194	160	536	306
12	15	29	137	106	332	207
13	12	26	169	121	507	164
14	11	22	120	119	410	325
15	18	27	138	130	398	365
16	12	35	191	157	504	376
17	15	16	169	136	424	400
18	16	30	180	113	348	230
19	22	56	237	241	731	985
20	23	35	333	318	972	1062
21	17	56	272	248	701	570
22	18	30	112	108	293	292
23	7	30	117	109	222	271
Total	451	821	5299	4532	13859	16360

3. A glance at the totals at the foot of the columns will show the marked irregularities of the scale. The number of stars of each magnitude should steadily increase in the ratio of about 3 : but there are actually fewer stars between mags. 9·0 and 9·9 than between 8·0 and 8·9. Forming N′, the sum up to and including the successive magnitudes, and then log N′, and subtracting 2·927 (as explained in the last note) to obtain the log N of Kapteyn, we have—

TABLE II.

Comparison with Kapteyn.

Mag.	N′.	Log N′.	Log N.	K_0^{90}.	K_0^{20}.	A − K_1.	A − K_2.
6·9	451	2·654	9·727	9·511	9·645	+0·216	+0·082
7·9	1272	3·104	0·177	0·000	0·149	+0·177	+0·028
8·9	6571	3·818	0·891	0·481	0·646	+0·410	+0·245
9·9	11103	4·045	1·118	0·950	1·139	+0·168	−0·021
10·9	24962	4·398	1·471	1·409	1·621	+0·062	−0·150
11·9	41322	4·616	1·689	1·856	2·091	−0·167	−0·402

4. Under the column K_0^{90} is given Kapteyn's general result for all galactic latitudes, and the excess of the Algiers result is given under the heading A − K_1. It will be seen that this is considerable until we reach magnitude 10·5; and this was the same for Bordeaux, though there was no sudden jump at magnitude 8·9 as in Algiers. We can transform this table as in the last paper so as to give the error of Algiers in magnitudes; see Table III.

TABLE III.

Kapteyn's Magnitudes from Algiers Counts.

Alg. Mag.	Log N.	K_0^{90}.	A − K.	B − K.
6·9	9·727	7·33	−0·43	−0·50
7·9	0·177	8·27	−0·37	−0·62
8·9	0·891	9·78	−0·88	−0·84
9·9	1·118	10·28	−0·38	−0·88
10·9	1·471	11·03	−0·13	−0·71
11·9	1·689	11·52	+0·38	−0·20

In the last column are added the corresponding results for Bordeaux, which are not dissimilar in some respects, but have no sudden jump at mag. 8·9 as above remarked. It is clear that there is some considerable error in the Algiers scale of magnitudes which can only be investigated at the observatory, and which need not wait for investigation until the more elaborate scheme for determining these scale errors by photographs of the Pole, etc., shall have been carried out.

5. We proceed to analyse the results harmonically in order of

R.A. For this purpose the numbers for 23^h and 0^h were taken together, for 1^h and 2^h, etc. etc. This combination of two consecutive groups gives us twelve points on the curve, and has the following advantages:—

(*a*) The analysis of twelve points is very simple and the points chosen are convenient, referring exactly to $0^h{\cdot}0$, $2^h{\cdot}0$, etc.

(*b*) Since the numbers of plates falling under hourly headings are alternately 7 and 8, the addition of two consecutive totals gives us a uniform series with fifteen plates in every group.

The resulting analytical expressions for log N′ (only differing from Kapteyn's log N by a constant, which does not affect the analysis) are as in Table IV.

TABLE IV.

Coefficients of First and Second Harmonics for log N′ for Algiers Zone $+1°$.

Alg. Mag.	K_0^{90}.	Sin α.	Cos α.	Sin 2α.	Cos 2α.	Combined.
6·9	7·3	+·08	−·09	−·14	−·20	$+{\cdot}12\cos(\alpha-138^\circ)+{\cdot}24\cos 2(\alpha-108^\circ)$
7·9	8·3	+·03	−·15	−·21	−·15	$+{\cdot}15\cos(\alpha-169)+{\cdot}26\cos 2(\alpha-117)$
8·9	9·8	+·08	−·08	−·18	−·19	$+{\cdot}11\cos(\alpha-135)+{\cdot}26\cos 2(\alpha-112)$
9·9	10·3	+·08	−·06	−·17	−·18	$+{\cdot}10\cos(\alpha-127)+{\cdot}25\cos 2(\alpha-112)$
10·9	11·0	+·08	−·07	−·16	−·18	$+{\cdot}11\cos(\alpha-131)+{\cdot}24\cos 2(\alpha-111)$
11·9	11·5	+·12	−·05	−·18	−·21	$+{\cdot}13\cos(\alpha-113)+{\cdot}28\cos 2(\alpha-111)$
Gron. No. 18	7·0	·00	·00	−·05	−·14	·00	$+{\cdot}15\cos 2(\alpha-100)$
	12·0	·00	·00	−·10	−·28	·00	$+{\cdot}30\cos 2(\alpha-100)$

6. At the foot of the table are given the similar analyses for Kapteyn's values of log N given in the *Pub. Ast. Lab. Groningen*, No. 18, and it will be seen that there are the following essential differences between Kapteyn's figures and the Algiers results:—

(A) Kapteyn has no sensible first harmonic, owing to the fact that he takes no note of difference of distribution in galactic longitude. (The equator cuts the Galaxy symmetrically, and as the Algiers zone is close to the equator, the absence of the first harmonic is a natural consequence of the distribution in galactic latitude only.) But the Algiers results show a very sensible first harmonic, confirming the counts from Stratonoff's maps (Argelander's zones), shown on p. 476 of this volume. We shall presently return to this point.

(B) The second harmonic, which shows more particularly the effect of galactic latitude, differs sensibly from Kapteyn's figures. Firstly the epoch is sensibly different, which may be due to the neglect of distribution in longitude; but, secondly (which is far more important), there is no sensible change of either epoch or coefficient in the Algiers results, while Kapteyn would lead us to expect a steady increase in the coefficient. Thus we have again evidence against the relative increase of faint stars as we approach

the Galaxy; such as was offered in *Mon. Not.*, lxix. p. 402, from counts on the Oxford, Potsdam, Toulouse, and Helsingfors plates, though from perhaps scanty material; was forthcoming from the counts of a complete zone of the Bordeaux plates (*Mon. Not.*, lxxii. p. 473); and is now a very plain outcome of counts of a wholly independent zone of the Algiers plates. Is not this weight of evidence becoming worthy of serious consideration?

7. For the sake of clearness it may be repeated here that this evidence offers no reason whatever why Kapteyn's figures should not be essentially correct. His figures refer to *visual* magnitudes, and the present figures to *photographic* magnitudes. The explanation of the discrepancy adopted from the first (see *Mon. Not.*, lxix. pp. 399 and 402) is that we have here a differential effect of the scattering of light in its passage to us: so that distant stars become photographically fainter. Stars of a given magnitude (say 9·0) are made up partly of brilliant stars at a distance and partly of dimmer stars near us. In the direction of the Galaxy, no doubt, the proportion of these brilliant and distant stars increases, so that the stars of a given magnitude are on the whole more distant when we look in the direction of the Galaxy. *Visually*, Kapteyn finds them relatively more numerous as we proceed to fainter magnitudes, and if *photographically* they are not so, is not this a perfectly legitimate consequence of their greater distance?

8. Further, there is no necessary inconsistency between the following two apparently irreconcilable statements:—

(α) The stars of the Galaxy are *bluer* in colour than those near its poles (Kapteyn).

(β) The distant stars of the Galaxy are *redder* than those near its poles (Turner).

For the former statement, which has been familiar to us at any rate since Kapteyn's Memoir of 1893 (*Bull. du Com. Int. Perm.*, Tome II.), applies to stars generally from the 4th magnitude to the 9th; for simplicity let us regard the mean star as of the 7th magnitude. Then at this magnitude Kapteyn's (*Gron. Pub.*, No. 18) visual effect of the Galaxy is represented by the coefficient 0·15 in log N: and the Algiers zones (*photographic*) coefficient 0·24, which is *greater*. Hence, in the Galaxy the 7·0 mag. stars are bluer. But the point is that this does not continue to hold for fainter magnitudes, because Kapteyn's *visual* coefficient is steadily rising while the *photographic* coefficient remains constant. The former thus passes the latter about magnitude 10·0 and is afterwards in excess. The more distant stars thus become redder. We may quite reasonably ascribe the former blueness to a difference in physical constitution and the gradual change to red to an increase of distance, without inconsistency.

9. We now return to the first harmonic of which the coefficient is steady though the epoch shows some change. The particular change shown is for the point of maximum to approach the 94° vertex as we take in fainter stars. But the discussion will be easier if, instead of gathering all magnitudes together under log

N′, we take each magnitude separately. We therefore return to the separate counts.

TABLE V.

First Harmonic for Separate Magnitudes.

Mag.	Sin θ.	Cos θ	Combined.
Brighter than 7·0	+3·4	−1·4	3·7 cos (α − 113°)
7·0– 7·9	+1·4	−2·2	2·6 cos (α − 148°)
8·0– 8·9	+3·5	−1·6	3·8 cos (α − 114°)
9·0– 9·9	+3·2	−1·0	3·4 cos (α − 107°)
10·0–10·9	+2·9	−1·8	3·4 cos (α − 122°)
11·0–11·9	+6·6	−1·1	6·7 cos (α − 99°)

We may repeat for comparison the figures derived from Stratonoff's maps and the Algiers charts (*Mon. Not.*, lxxii. p. 476).

TABLE VI.

First Harmonic from Stratonoff's Zones (−5° to +5°).

Mag.	First Harmonic.
To 6·0	1·3 cos (α − 36°)
6·1 to 6·5	1·3 cos (α − 29°)
6·6 to 7·0	1·2 cos (α − 273°)
7·1 to 7·5	0·3 cos (α − 108°)
7·6 to 8·0	0·8 cos (α − 73°)
8·1 to 8·5	1·8 cos (α − 79°)
8·6 to 9·0	2·1 cos (α − 80°)
9·1 to 9·5	1·1 cos (α − 77°)
Algiers Charts	2·2 cos (α − 52°)

The Algiers Catalogue counts give a considerably higher coefficient (on the scale of 10 for the average density) and the epoch is on the *other side* of the 94° vertex. But it is noteworthy that as we come to fainter stars there is a distinct tendency towards the vertex in both series. It is clear that more evidence would be very welcome, but what we have is not unfavourable to the conclusion that there are more faint stars in the direction of the 94° vertex, as demanded by the hypothesis of a finite stellar system. (*Mon. Not.*, lxxii. p. 476).

Summary.

§ 1. Further recurrence to the simple proposal made at Paris in 1909; and to the criterion for distinguishing between the "In Vertex" and "Out Vertex" of our sidereal system.

§ 2. Summary of counts for Algiers (Dec. + 1°) in hours of R.A. (Table I.).

§ 3, 4. Approximate error of Algiers magnitude scale ranging from −0·78 to +0·38 of a magnitude according to Kapteyn (Table III.). There is also a sudden jump in the Algiers scale about magnitude 9·0.

§ 5. Harmonic analysis of the counts compared with Kapteyn's figures from distribution in galactic latitude

§ 6. There is a sensible first harmonic, not derivable from Kapteyn, who takes no account of galactic longitude. And there is no progressive increase in the second harmonic.

§ 7. Recurrence to the conclusion, already supported by much evidence, that the light of Milky Way stars becomes photographically faint as we go to fainter magnitudes, owing to their greater distance and the consequent differential arrest of the light from them. But this does not contradict Kapteyn's conclusion that for the stars between magnitudes 4·0 and 9·0 (say), the Milky Way stars are bluer (photographically brighter).

§ 8. Discussion of the first harmonic which supports the conclusion that faint stars increase towards the "In Vertex" (94° + 12°).

Micrometrical Measures of Double Stars

By Rev. T. E. R. Phillips, M.A.

The instruments employed in making the following measures were: (1) a 12¼-inch reflector by Calver; and (2) the Coleman refracting telescope of 8 inches' aperture by Cooke, recently lent by the Council of the R.A.S. The latter was used in making the measures of the present year. The reflector is now employed in the measurement of faint stars demanding a considerable grasp of light.

The places are those given by Mr. Lewis (*Mem. R.A.S.*, vol. lvi., 1906), corrected for precession, and the magnitudes are those of Struve.

Star.	1910 R.A.	Dec.	P.	D.	Mags.	Nights.	Date.
	h m s	° ′	°	″			
716	5 23·5	+25 5	201·6	4·73	5·8, 6·6	3	1911·090
1110 (Castor)	7 28·9	32 6	220·9	5·49	2·7, 3·7	3	1912·060
1126	35·3	5 27	145·5	1·06	7·2, 7·5	4	1911·088
1129	36·7	18 17	63·5	21·65	8·2, 8·7	3	1911·090
1130	36·8	9 55	173·3	1·15	8·4, 8·9	4	1911·088
1137	41·8	4 22	131·9	3·08	8·0, 9·0	3	1911·090
1140	43·1	18 34	273·8	6·31	6·8, 8·5	3	1911·095
1196 (ζ Cancri) AB	8 7·1	17 55	326·2	0·99	A+B=5	3	1911·110
,, AC	...	...	110·8	4·96	C=5·7	3	1911·110

Star.	1910. R.A.	Dec.	P	D.	Mags.	Nights.	Date.
	h m s	° ′	°	″			
Σ 1202	8·6	11 7	315·0	2·19	7·7, 9·8	3	1911·17
,, 1206	9·9	7 21	200·6	12·62	9·0, 9·5	2	1911·08
,, 1224	21·3	24 50	44·2	5·87	6·0, 7·1	3	1911·17
,, 1228	22·2	27 51	351·3	9·27	8·0, 8·5	3	1911·16
,, 1273 (ϵ Hydræ $\frac{AB}{2}$ and C)	42·0	6 45	240·5	3·49	A = 3·8	2	1911·20
,, 1424 (γ Leonis)	10 15·0	20 18	118·3	3·81	2·0, 3·5	6	1911·67
,, 1431	20·9	9 14	70·2	3·33	7·5, 9·2	3	1912·29
,, 1434	22·3	18 31	274·1	6·29	8·5, 8·5	3	1912·35
,, 1439	25·1	21 16	112·9	1·63	8·0, 8·5	3	1912·35
,, 1450	30·3	9 7	155·9	2·14	6·0, 8·7	3	1912·36
,, 1596	11 59·7	21 58	238·2	3·77	6·0, 7·5	3	1912·21
,, 1669	12 36·6	− 12 31	306·5	5·74	6·5, 6·5	3	1912·19
,, 1670 (γ Virginis)	37·0	0 57	323·6	5·95	3·0, 3·0	3	1912·23
,, 1681	45·1	+ 4 19	195·1	8·76	8·5, 8·5	3	1912·36
,, 1687	48·9	21 44	90·2	1·22	5·2, 8·0	3	1912·35
,, 1744 (ζ Ursæ Maj.)	13 20·4	55 24	149·5	14·42	2·1, 4·2	3	1912·29
,, 1755	28·4	37 17	130·7	4·26	7·0, 7·9	2	1912·21
,, 1756	29·0	23 29	177·5	14·43	8·5, 9·0	2	1912·21
,, 1825	14 12·4	20 32	171·1	4·22	6·8, 8·5	3	1912·36
,, 1864 (π Boötis)	36·5	16 47	103·4	5·72	4·9, 6·0	3	1912·29
,, 1877 (ϵ Boötis)	41·1	27 27	330·3	2·94	3·0, 6·3	5	1911·83
,, 1884	44·5	24 44	54·3	1·68	6·2, 7·8	3	1912·36
,, 1888 (ξ Boötis)	47·2	19 29	128·4	2·05	4·7, 6·6	4	1912·36
,, 1937 (η Coronæ Bor.)	15 19·5	30 37	43·9	1·12	5·5, 6·0	3	1912·42
,, 1974	44·5	− 2 58	162·2	2·53	8·5, 8·7	3	1912·36
,, 1985	51·2	1 54	339·0	5·89	7·0, 8·1	3	1912·36
,, 2021	16 9·1	+ 13 46	336·6	4·03	6·7, 6·9	3	1912·41
,, 2032 (σ Coronæ)	11·3	34 4	216·8	4·88	5·0, 6·1	3	1912·41
,, 2052	24·9	18 36	81·5	0·93	7·5, 7·5	3	1912·41
,, 2055 (λ Ophiuchi)	26·3	2 11	68·1	1·16	4·0, 6·1	3	1912·39
,, 2094	40·4	23 41	78·2	1·48	7·3, 7·6	3	1912·41
,, 2114	57·7	8 35	165·5	1·27	6·2, 7·4	3	1912·41
,, 2140 (α Herculis)	17 10·5	14 29	113·5	4 60	3·0, 6·1	4	1911·32

Star.	1910 R.A.	Dec.	P.	D.	Mags.	Nights.	Date.
	h m s	° ′	°	″			
Σ 2161 (ρ Herculis)	20 0·6	37 13	314·9	3·89	4·6, 5·5	3	1912·423
,, 2382 (ϵ^1 Lyræ)	18 41·3	39 35	11·2	2·95	4·6, 6·3	3	1911·437
,, 2383 (ϵ^2 Lyræ)	41·4	39 31	123·8	2·30	4·9, 5·2	3	1911·437
,, 2628	20 3·4	9 8	345·3	3·98	6·1, 8·2	3	1911·602
,, 2634	5·4	16 32	11·9	5·44	8·0, 9·5	3	1911·602
,, 2641	7·4	3 32	169·8	20·83	7·5, 11·2	3	1911·602
,, 2881	22 10·4	29 8	98·9	1·48	7·7, 8·2	4	1911·650
,, 2895	16·5	24 29	35·3	8·10	8·5, 10·0	2	1911·723

Notes.

Σ 1130. The published measures of this star are discordant, especially those of angle. The distance is diminishing.

Σ 1424 and Σ 1877. Means of measures in 1911 and 1912.

Σ 2052. Now closing rather rapidly.

Ashstead, Surrey,
June 1912.

Two probably Variable Stars. By the Rev. T. E. Espin, M.A.

No. 1. The star B.D. + 46° 1635, R.A. $10^h\ 22^m\ 47^s{\cdot}4$, Decl. + 46° 15′·8 (1855), Mag. 9·3. On April 2 it had a magnitude about 9·0. On subsequent nights it was compared with B.D. + 46° 1637, Mag. 8·8 and B.D. + 46° 1643, Mag. 8·4 with the following results:—

1912 April 8	8·4
10	8·4
11	8·4
16	8·4

The colour is pale orange-red, and the spectrum type III.

No. 2. A star which is not in the B.D. Its place from B.D. + 44° 2265 = A.G. (Bonn) 8924 is—

R.A. $13^h\ 16^m\ 56^s{\cdot}3$. Decl. + 44° 30 42″·9 (1900)

The observations are—

1912 May 13	9·2	Red, Type IV.
18	9·5	Red, Type IV.

The star B.D. + 44° 2267 was found to be missing on both nights, and it may be identical with this star, if the R.A. in the B.D. is increased by 5 seconds and the Declination by 2′.

Nova 2 Geminorum. By the Rev. T. E. Espin, M.A.

Estimates of the magnitude were made with a pair of field-glasses during the brighter phase, subsequently with a 3-in. achromatic. The following comparison stars were used :—

θ Geminorum	Mag.	3·7
τ ,,		4·6
B.D. +32°·1414		5·7
B.D. +32°·1433		6·9
B.D. +32°·1437		8·2

1912 March	14	9.0 p.m.	4·0	1912 April	14	10.45 p.m.	6·8
	16	11.50 ,,	4·8		16	9.30 ,,	6·8
	17	8.0 ,,	5·2		19	9.45 ,,	6·7
	19	7.15 ,,	5·4		20	8.45 ,,	7·0
	20	8.0 ,,	5·2		21	9.0 ,,	6·9
	21	9.45 ,,	5·5		22	9.0 ,,	7·1
	22	8.0 ,,	5·2		23	9.0 ,,	7·0
	28	8.45 ,,	5·9		24	9.0 ,,	7·1
	29	10.45 ,,	6·0		30	9.30 ,,	6·8
April	1	8.15 ,,	6·1	May	2	9.30 ,,	6·9
	2	8.30 ,,	6·0		11	10.20 ,,	7·3
	6	8.15 ,,	6·5		12	9.45 ,,	7·3
	8	8.15 ,,	6·5		13	9.40 ,,	7·3
	10	8.30 ,,	6·7		18	10.0 ,,	7·3
	11	8.15 ,,	6·7				

The following are the observations of colour with the $17\frac{1}{4}$-in.:—

1912 March	16	Yellow.	1912 April	1	Fine red.
	19	Orange.		2	Fine red.
	20	Deep yellow.		6	Fine red.
	22	Deep orange.		11	Very red.
	28	Fine, but not deep red.	May	2	Orange (3-in. refractor).

Spectroscopic Observations.

It is needless to quote these in detail. On March 16 the spectrum was continuous, with bright hydrogen lines. On March 19 the spectrum had quite changed, and had become banded. The nebula line was seen. March 20, C and F were equally bright. March 28, C extremely brilliant. April 1, C is now extraordinarily brilliant, if $\lambda 500 = 0$, F = 7, C = 10. Similar observations on April 2 and 6. April 11, the fading of the

continuous spectrum makes the bright lines stand out more prominently.

General conclusions:—

(1) The Nova has been singularly unstable in light.

(2) The colour has changed from yellow to very red, and was evidently at the end of the observations returning to yellow.

(3) The colour-change is shown by the spectroscope to be due: (1) to the appearance of bands; (2) to the variation in the intensity of the hydrogen line C, the deepest tint of red corresponding with the brightest phase of the C-line.

Observations of the Spectrum of Nova Geminorum.
By L. Becker, Ph.D.

1. The spectrum of the Nova was photographed on March 15, with the spectrograph attached to the 20-inch reflector of Glasgow Observatory. The 60° prism measures $6\frac{1}{2}$ inches on a side; the aperture is $3\frac{1}{4}$ inches; and the focal length of the camera, 59 inches, is about twice that of the collimator. The spectrograph is automatically kept at a constant temperature, and the effect of flexure is eliminated by frequent adjustment of the plate on the D-lines. By means of a curved film-holder the spectrum is in focus on the film over its whole range. The width of the slit was 0·05 mm. The iron-sodium arc furnished the comparison spectrum. The film was sensitised for the red-yellow-green region. The spectrum of the Nova extends from 684 to 406, occupying 2·8 inches, while the comparison spectrum has been photographed to 369. The film was measured four times, twice in each direction with a low and a medium magnifying power. The reduction of the measurements to wave-lengths was obtained from a table which is based on Ketteler's formula of dispersion, the corrections to the tabular values being determined graphically from the measurements of the comparison lines. Each of the four sets was reduced separately. Their mean values appear in column 2 of the table. Column 1 gives the grades of intensity, the notation 2–4 indicating that the intensity increases from grade 2 to 4. The position of an absorption line is shown by its intensity being printed in italics.

2. I have shown in my paper on Nova Persei * that the structure of the bands into which the spectral lines are broadened is similar, and that the broadening is proportional to the wave-length and independent of the element. On the assumption that this relation also applies in the present case, the observations of the hydrogen lines and the lines 4922 and 5016 give for the absorption line a displacement of -9 *t.m.*; the maximum intensity

* *Trans. R.S. Edin.*, vol. xli.

extends from −4 to +10 *t.m.* and fades to −6 and +21 *t.m.* The figures, however, are uncertain owing to the diffuse character of the bands. There is no decisive evidence that each band has more than one absorption line. As neighbouring lines displace the positions of the maxima of their bands, the most reliable determination of the wave-length of the spectral line to which the band belongs will be obtained from the absorption line. These wave-lengths (wave-lengths of the absorption lines multiplied by $1+8{\cdot}5/4500$) are given in column 3 of the table. The hydrogen lines H_α to H_δ leave the residuals +1·1, −1·5, −1·3, +0·5 respectively—a discordance which must be ascribed to the difficulty in selecting corresponding points in these ill-defined absorption bands. Most of the other absorption lines have a breadth of 2 to 4 *t.m.* and are well defined, so that the figures will be more accurate than those for the hydrogen lines. All the lines recorded in the spectrum of Nova Aurigæ are present except 4737 (Campbell), 4125 (Campbell and Vogel), and some in the red-yellow region. The wave-lengths of the lines not observed before are printed in italics in column 3. The line 5894 is the sodium line and certainly not the D_3 line of helium.

3. The spectrum of the Nova was further photographed on seven nights, March 15 to April 11, with a 15° prism placed in front of a $3\frac{1}{4}$-inch lens of 30 inches focal length. This prismatic camera is attached to the tube of the 20-inch reflector, and it is guided by holding the star on the slit of the large spectrograph. The spectrum occupies 0·5 inch from 0·37 to 0·68. As to the reductions, the four constants of Ketteler's formula were determined from photographs of α Lyræ, and the screw-reading of the measuring apparatus was tabulated as a function of the wave-length.

The first photograph was secured on March 15, simultaneously with the one treated above. The estimated intensities are given in column 4. It will be seen that in the red and violet regions they differ greatly from those in column 1, the disagreement being due to the different action of the sensitising solution on different kinds of plates. Much of the detail shown in the large photograph is obscured, especially between H_β and H_γ, where there are only 7 absorption lines against 25 revealed by the spectrograph. Additional bands in the violet region are given at the end of the table.

The intensities on April 9 appear in column 5 on the lines where there were maxima on March 15, and where a band in the April spectrum covers several maxima recorded on March 15 the intensity is repeated. These and other photographs taken on March 20, April 1, 2, 10, 11 give the following results:—

The continuous spectrum is absent on the plates, except on March 15. On March 20 the plate contains 20 absorption lines, and there is none on the later plates. The hydrogen bands altered in intensity and breadth; relatively to H_γ the less refrangible ones increased in brightness, H_α being on April 9 and 11 the brightest line of the spectrum. On March 15 H_β and H_γ ex-

tend over 22 *t.m.*, on March 20 over 16 *t.m.*, on April 1 and 2 over 25 *t.m.* with a faint extension towards the more refrangible side, and on April 9, 10, 11 over 21 *t.m.* Almost all the other bands faded in that period relatively to H_γ, and some vanished, as shown by the figures given in the last two columns. A remarkable feature is the persistence in brightness of the band belonging to the lines 4629 and 4670. The spectrum of Nova Persei showed the same peculiarity in March 1901, when the bands which I ascribed to the lines 4637 and 4687 greatly increased in intensity (*l.c.*).

4. *Contents of the Table.*—Columns 1 and 2 give the intensity curve recorded with the spectrograph on March 15 (see section 1): *e.g.* the intensity increases from grade 5 at wave-length 6585 *t.m.* to 9 at 6575·7, and is 10 from 6575·7 to 6558·5, etc. An absorption line is indicated by its intensity being printed in italics. The figures from 4050 to 3845 belong to the photograph taken on March 15 with the prismatic camera, and the intensities are entered in column 4. From 3980 to 3920 the wave-lengths belong to points at which the intensities change, while from 4050 to 3990 and from 3914 to 3871 the wave-lengths of the maxima and minima are given.

Column 3 contains the wave-lengths of the spectral lines to which the bands given in column 2 belong (see sections 2 and 3).

Columns 4 and 5 contain the intensities recorded with the prismatic camera respectively on March 15 and April 9 (see section 3).

TABLE.

1	2	3	4	5
	6817			
2–4	6585			
5–9	6575·7			
10	6558·5		1	13
9–4	6554·0	6563·9		
0	6549·0			
1–3	6536			
5	6442	?	0	0
5–1	624			
1–3	5926	*593*	0	0
1?	5914			
3	5906·5			
5	5895·3		0	0·5
3	5889·9	5894		
0	5875·5			
2	5871			
4	5799			
3–2				

1	2	3	4	5
	570			
2–3	5637	?		
4	5577		1	0·5
3–2	5564			
2–3	5553			
4	5518	5527	1	0
0?	5517			
3	5308	5318		
0?	5307			
4	5273·0	5278·9	0	0
0	5264·8			
2	5252·8			
4	5225·0	5228·3	0	
1	5211·8			
4	5165·2	5171	1	0·5
0?	5157			
2	5152	5157		
0?				

TABLE—*continued.*

1	2	3	4	5
	5142·6			
3	5125·5	*5128*	0	0
1?	5111			
2 – 3	5068			
4	5046	5052·1	0	0·5
0	5039			
5	5031·5	*5038·0*	0	0·5
0	5025·5			
5	5012·6		3	0·5
3	5010·7	5015·8		
0	5001·9			
2 – 3	4999·0			
4	4957	*4959*	4	0·5
3?	4943			
4	4934·0			
6	4919·4			
5 – 4	4915·4	4922·5	5	1
0	4911·0			
5	4892·6	*4900·7*	4	0
4	4890·4			
5	4885·0			
8	4876·6	*4884·3*	10	12
5	4873·7			
9 – 10	4866·8		10	12
12	4857·2		10	12
8 – 4	4854·5			
0	4847·1	4860·0		
4 – 6	4837·5			
7	4817·3	*4825·0*	8	0
4	4814·5			
6	4797·7	*4805·3*	8	0
4	4794·7			
6 – 7	4780			
7 – 6	4765·2	4772·5	9	1
4	4761·8			
6	4755·3	*4762·4*	8	1
4	4751·5			
6 – 7	4715·2	?	8	1
7 – 6	4677		8	9
7				

1	2	3	4	5
	4661·8	4669·7	8	9
5	4660·0			
7	4638·8			
9	4627·0		10	9
9 – 7	4622·4	4629·5		
5	4619·2			
8	4611·9	*4618·6*	7	4
4	4607·8			
6 – 8	4598·3			
9	4584·1		10	4
6	4575·5	4582·8		
4	4573·0			
8	4563·9	4571·3	6	4
4	4561·5			
8	4556·1	4563·8	6	4
3	4554·3			
8	4548·6	4556·0	6	7
4	4546·3			
7	4541·9		7	7
3	4539·0	4549·1		
8 – 9	4529·2		7	7
9 – 8	4527·1	4533·8		
4	4523·3			
8	4514·7	4521·9	8	5
5	4512·1			
8	4501·3	4508·8	7	5
6	4499·3			
9	4493 5	4500·7	7	5
4	4491·0			
9	4481·1	4488·9	7	5
6	4478·4			
8	4474·7	4481·8	7	5
6	4472·0			
8	4463·1		7	7
7 – 5	4461·1	4468·3		
4	4458·7			
5 – 7	4455·8			
8	4442 2		8	7
7 – 6	4436·9	4443·7		
4				

TABLE—*continued.*

1	2	3	4	5
6 - 7	4433·8			
8	4429·8			
7 – 6	4413·4		8	6
4	4410·0	4416·8		
6 – 7	4407·0			
7 – 8	4405·6			
8	4404·3			
7 – 6	4395·3		8	6
5 – 4	4391·8			
3	4387·7	*4394·4*		
7	4384·5			
5	4378·0	4384·6	8	6
7	4374·6			
6 – 5	4369·1		8	6
3	4367·9	4374·5		
5 – 6	4364·7			
7	4358·0			
8	4351·0		12	12
4	4345·8	*4352·8*	12	12
6 – 7	4343·4			
8	4342·0			
6 – 5	4335·6		12	12
2	4333·6	4339·3		
4 – 5	4328·6			
6	4322·7			
5	4316·5		8	5
2	4307·6	4314·3		
5	4304·8			
2	4295·1	*4301·2*	8	6
4	4291·2			
4 – 3	4286·9		8	5
1	4284·1	4290·6		
2 – 3	4281·0			
4	4278·8			
3 – 2	4267·4		9	4
1	4266·2	4272·5		
2 – 3	4262·6			
4	4260·3			
3 – 2	4245·1		9	4

1	2	3	4	5
1	4240·5	4247·4		
4	4238·3			
1	4228·2	4234·6	8	5
2	4225·1			
0?	4219	4226		
2	4217			
0	4209·6	4215·1	8	4
1 – 2	4204 8			
3	4194·0			
2 – 1	4176·0		9	5
0	4174·3	4179·1		
2	4168·1			
2 – 1	4161·5		8	5
0	4158·9	4164·3		
1 – 2	4154·1			
3	4111·9			
2	4098·7		11	9
0	4096·6	4102·5	11	9
1 - 2	4093·1			
2	4085·5			
0	4073·4	4078·9	10	0
1	4069·0			
	4050		8	4
	4040	4048	*5*	
	4029		8	3
	4023	4031	*5*	
	4003		8	1
	3990	3998	*4*	
	3980		11	6
	3970		8 – 6	
	3965	3968	*0*	
	3956		6	2
	3947	3953	*0*	
	3944		12	2
	3936		8 – 6	2
	3930	3932	*0*	
	3920			

TABLE—*continued.*

1	2	3	4	5	1	2	3	4	5
	3914		4	2		3887		3	4
	3906	3913	*0*			3871	3878	*0*	
	3902		4	2		3867			
	3895	3902	*0*			3845		2	0

Glasgow University Observatory:
1912 *May* 31.

Note on the Spectrum of Nova Geminorum 2. 1912 *April* 24.
By the Rev. A. L. Cortie, S.J.

A good spectrogram of the star was obtained with the Hilger single compound prism attached to the 15-inch equatorial on 1912 April 24. The magnitude of the star was 6·80 (from information kindly supplied by Dr. Rambaut), and the exposure on an Edwards' Isochromatic plate was nearly two hours with a very slow trail. This spectrogram is interesting as representing a transitional phase in the successive spectral stages of the star. Pending a full discussion of all the plates secured, the present note gives the chief characteristics of this single spectrogram. Visually, the star was of a brick red tint, and was apparently brighter than it had been two days previously.

In the earlier days of April the prominent band, about mean wave-length 4650, had been of less intensity than the complex hydrogen lines H_β and H_γ. On April 10 and April 12 this band was of less intensity than H_β but brighter than H_γ. On April 23 and April 24 it had increased in brightness until it equalled, if it did not surpass H_β. So that, in the present case, as in Nova Persei the temporary brightening was due to an increased brilliancy of a band spectrum, accompanied by a decrease in the brilliancy of the hydrogen lines, or series of lines, for H_β, H_γ, and H_δ each had a structure consisting of three lines. No dark lines are unmistakably recognisable in the spectrogram, though one or two are suspected. The spectrum consists of a series of bright bands, and of bright lines. Moreover, in each bright band there are three or more prominent bright lines, either bright lines under superposed bright patches of continuous spectrum, or, what is more probable, constituents of a series of flutings. The bands are generally sharp on the more refrangible edges, and hazy on the less refrangible sides. There are sixteen such bands between wave-lengths 5788 and 4097, including the three hydrogen complexes. There are 102 bright lines in all, including those in the bands. Of these lines 46 coincide in position with bright lines observed by Father Sidgreaves,* and Professor Campbell †

* *Memoirs R.A.S.*, li. pp. 36–43.
† *Ast. and Ast. Phys.*, No. 109, Nov. 1892.

in the spectrum of Nova Aurigæ; 57 are coincident with chromospheric lines occurring in the lists of Dr. Dyson,* and 54 are enhanced metallic lines which occur in Sir Norman Lockyer's *Tables of Wave-Lengths of Enhanced Lines.* Among the metals represented are iron, titanium, chromium, manganese, nickel, vanadium, zirconium. So far it has not been possible to ascertain the origin of the banded spectrum. But the presence of so many chromospheric and enhanced lines might possibly, though not necessarily, be an evidence of an origin of the Nova due to an outburst, similar to such as are witnessed in solar eruptions, though on a vastly superior scale. The helium lines 4472, 4713, are not apparent, though there are lines in approximate coincidence with those at 4922 and 5016.

Stonyhurst College Observatory:
1912 *June* 11.

Further Observations of the New Star in Gemini, No. 2, made at the Radcliffe Observatory, Oxford.

(*Communicated by Dr. A. A. Rambaut, M.A., F.R.S., Radcliffe Observer.*)

This paper is a continuation of one published in the *Monthly Notices* for April (pp. 506–511). It contains the results of the observations of the magnitude of Nova Geminorum, No. 2, made at the Radcliffe Observatory from April 11 to June 2.

The estimates of brightness have all been made with telescopic aid, the instruments employed being the 2·7-inch finder (power 13) of the Barclay equatorial, the 1·9-inch finder (power 17) formerly attached to the heliometer, and a 1·2-inch telescope (power 8). The 10-inch (power 85) was used on a few occasions for observations on the character and colour of the image of the Nova.

In Table I. are given the reference letters, the names of the stars used for comparison, and the Harvard magnitudes on which the estimates are based.

Table II. contains the separate estimations and the mean magnitudes corresponding to the G.M.T.'s given in the first column.

Three methods of estimating the brightness have been used. The first is Argelander's well-known method of steps, adopted by Mr. Barrett only, the value of the step being determined as $0^{m}\cdot045$; the second, the proportional method, in which the variable is compared with two stars, one brighter and one fainter, and an estimate is made of the ratio which the difference between each comparison-star and the variable bears to the difference between the two comparison-stars; and the third, the direct method of approximate estimation with the unit as one magnitude.

In this table the letter N represents the Nova.

* *Memoirs R.A.S.*, lvii. Appendix.

It will be seen that the oscillations of brightness of the Nova, referred to in the earlier paper, persisted for a short time, but with decreasing amplitude. After June 2 observations were discontinued, estimations becoming too difficult, owing to encroaching twilight and diminishing altitude of the star.

The observers were:—

Mr. W. H. Robinson, indicated by R.
Mr. H. G. S. Barrett, „ „ H.B.

TABLE I.

List of Stars used for Comparison with Nova Geminorum, No. 2.

Ref.	Name of Star.	Harvard Photom. Mag.	Ref.	Name of Star.	Harvard Photom. Mag.
a	B.D. +32° 1414	5·76	*h*	B.D. +33° 1377	7·17
b	B.D. +33° 1433	6·01	*i*	B.D. +33° 1383	7·64
c	B.D. +32° 1460	6·46	*j*	B.D. +32° 1421	7·8†
d	B.D. +32° 1378	6·54*	*k*	B.D. +35° 1513	7·87
e	B.D. +32° 1500	6·66	*l*	B.D. +32° 1437	8·24‡
f	B.D. +32° 1433	6·89	*m*	B.D. +32° 1418	8·3†
g	B.D. +35° 1495	7·07			

TABLE II.

Estimations of Magnitude of Nova Geminorum, No. 2.

Date and G.M.T. of Observations.		Instrument.	Estimations of Magnitude.	Mean Mag.	Observer.
1912.	h m				
Apr. 11	8 15	Barclay finder	*a* 10 N 2*f*; N = *c*	6·61	R.
	8 20	„	N 5*f*, *c* 6 N	6·70	H.B.
12	8 0	„	*a* 7 N 3*c*, *a* 7 N 6*f*	6·31	R.
	8 17	„	*a* 3 N 2*f*, *b* 3 N 1*c*, *b* 5 N 5*f*; N 6*c*	6·36	H.B.
15	7 45	„	*c* 3 N 3*f*	6·68	R.
	9 4	„	*c* 7 N, N 3*f*	6·77	H.B.
	10 11	„	*c* 8 N, N 2·5*f*	6·80	H.B.
16	7 45	„	*a* 7 N 4*f*, *c* 3 N 4*f*	6·56	R.
	10 15	1·2-inch telescope	*c* 5 N, N 6*f*	6·65	H.B.
17	8 50	Barclay finder	*c* 5 N 5*f*	6·68	R.
	8 55	„	N 3·5*f*, *c* 6 N	6·73	H.B.

* From Radcliffe estimations on Harvard scale, obtained from the observations given in Table II. The magnitude given in the *Harvard Photom. D.M.* (vol. xlv.) is 6·87, and in the *Potsdam D.M.* (corrected to Harvard scale) 6·96.

† B.D. mag.

‡ Photom. mag. reduced to Harvard scale (Jost: *A.N.* 4574).

TABLE II.—*continued.*

Date and G.M.T. of Observations.		Instrument.	Estimations of Magnitude.	Mean Mag.	Observer.
1912.	h m				
April 19	8 0	Barclay finder	a 8 N 6 f, N 2 c 4 f	6·33	R.
	8 55	,,	c 1 N, N 6 f; b 5 N 4 f, a 3 N 2 f	6·50	H.B.
20	8 40	,,	c 3 N 2 f	6·72	R.
	8 47	,,	N 5 f, c 6 N	6·70	H.B.
22	8 0	,,	N = f; c 0^m·5 brighter than N	6·92	R.
	9 15	,,	N 1·5 f, d 5 N	6·80	H.B.
23	8 10	,,	c 4 N 2 f	6·75	R.
	9 8	1·2-inch telescope	c 6 N, N 3 f, d 4 N	6·73	H.B.
	9 36	,,	c 7 N, N 4 f, d 3 N	6·72	H.B.
24	8 15	Barclay finder	c 7 N 2 f	6·79	R.
	8 54	1·2-inch telescope	N 2 f, d 7 N	6·83	H.B.
	9 33	,,	N 2·5 f, d 7 N	6·82	H.B.
	9 57	,,	N 4 f, c 6·5 N, d 5 N	6·74	H.B.
	10 30	,,	N 1 f	6 84	H.B.
	10 48	,,	N 2 f, d 6 N	6·80	H.B.
25	8 41	,,	N 1 f	6·84	H.B.
	9 30	Barclay finder	f 0^m·1 brighter than N	6·99	R.
	9 31	,,	N 0·5 f	6·87	H.B.
	10 27	1·2-inch telescope	N = f	6·89	H.B.
26	8 15	Barclay finder	c 7 f 2 N	7·01	R.
	9 57	1·2-inch telescope	f 3·5 N	7·05	H.B.
27	9 6	Barclay finder	N 5·5 h ($\frac{1}{2}$-weight), f 1·5 N	6·95	H.B.
	9 41	1·2-inch telescope	f 2·5 N	7·00	H.B.
29	9 24	Barclay finder	N 1·5 f, d 6 N	6·81	H.B.
	9 49	1·2-inch telescope	N 2 f, d 5 N	6·79	H.B.
30	8 53	Barclay finder	f 1 N	6·94	H.B.
May 2	8 53	,,	N 1·5 f, e 3 N	6·81	H.B.
	9 6	,,	N 0·5 f, e 2·5 N, d 7 N	6·83	H.B.
8	9 16	,,	f 7 N, g 6 N; g 4 N 6 k	7·31	H.B.
14	9 45	Heliometer finder	f 6 N	7·16	H.B.
16	9 20	Barclay finder	f 5 N 8 l	7·42	R.
17	10 17	Heliometer finder	f 5 N 6 j, f 5 N 7 m, h 5 N 3 i	7·41	H.B.
June 2	10 20	,,	N 5 l	8·01	H.B.

Observers' Remarks.

1912.

April 11. $8^h\ 15^m$. The image of the Nova is larger but less luminous than that of *c*. (R.)

April 15. $7^h\ 45^m$. With the 10-inch, power 85, image of Nova yellow with deep red fringe at ordinary focus. The extra-focal image is ruby. Twilight field. (R.)

April 19. $8^h\ 0^m$. With 10-inch, power 85, extra-focal image of Nova ruby; deep red fringe at ordinary focus as before. (R.)

April 22. $8^h\ 0^m$. Colour of images as on the 19th. (R.)

April 25. $9^h\ 30^m$. Image of Nova larger than, but not so bright as *f*. (R.)

April 30. $8^h\ 53^m$. Sky hazy, but observation considered good. (H.B.)

May 14. $9^h\ 45^m$. Not a very satisfactory observation; sky too hazy. (H.B.)

June 3. $10^h\ 40^m$. Nova too faint for observation, only just perceptible, very low. (H.B.)

Radcliffe Observatory, Oxford:
1912 *June* 13.

The Absorption of Light in Space. (Second Paper.)

By Francis G. Brown.

Section I.—The Amount of Systematic Error in the N.G.C.

(1) In a former paper (*M.N.*, Jan. 1912) I directed attention to a method of determining the absorption of light by comparing the brightness of large and small nebulæ. Since there were apparently no systematic measures of brightness, I had to fall back on the information contained in the N.G.C., while pointing out that the results might be exaggerated by systematic errors in the descriptions of brightness. In this present section I have endeavoured to determine the probable amount of this systematic error by comparing the photometric measures of certain nebulæ with the descriptions of them in the N.G.C. If no systematic error exists, the mean *measured* brightness of large and of small nebulæ of the same order of brightness in the catalogue should be the same, and any residuals will represent the systematic errors.

(2) The measures used are taken from *Harvard Annals*,

vol. xxxiii., No. 7. This volume contains measures of some 42 objects, most of which are bright, but there are a few faint ones included. It might perhaps be wondered why these photometric observations could not be used to determine the absorption directly. But since the objects observed were selected generally on account of their brightness, it is clear that we cannot use them in a determination the success of which depends on the use of material selected entirely at random. We can, however, use the measures indirectly, by finding from them the errors in the catalogue and then applying the corrections to the general result.

(3) Thus in Table I. I have classified Professor Pickering's photometric measures under the headings of size and brightness by which the particular nebulæ examined were described in the N.G.C. The mean of each order of brightness is given at the foot of the corresponding columns.

TABLE I.

Mean Photometric Brightness corresponding to each order of size and brightness in the N.G.C.

	*v*B.	B.	*c*B.	*p*B.	*p*F.	F.	*v*F.
*v*L	10·0	...	...	...	10·8	...	...
L	6·8	11·1	...	...	...	10·1	12·5
*p*L	8·1	7·3	9·2	10·5	13·0	...	...
*p*S	5·9	8·9	...	10·3	10·9	11·2	...
S	5·5	9·0	11·0	9·1	...	11·2	...
*v*S	4·2	6·8	...	8·0	...	...	...
*e*S	...	...	...	(7·2)	...	...	...
Means	6·7	8·6	10·1	9·0	11·6	10·8	12·5

The residuals are given in Table II. But, instead of using the means as they stand, it would perhaps be better to determine a smoothed mean. On plotting the numbers it appears that a straight line does rough justice. We can therefore get the equation by an automatic process, thus :—

$$\frac{(12\cdot5 - 6\cdot7) \times 6 + (10\cdot8 - 8\cdot6) \times 4 + (11\cdot6 - 10\cdot1) \times 2}{6^2 + 4^2 + 2^2} = 0\cdot83.$$

The mean value at *p*B from mean of all is 9·9. Hence the theoretical means are :—

*v*B = 7·4	*p*F = 10·7
B = 8·2	F = 11·6
*c*B = 9·1	*v*F = 12·4
*p*B = 9·9	

TABLE II.

Residuals from Smoothed Means of each order of Brightness.

	*v*B.	B.	*c*B.	*p*B.	*p*F.	F.	Means.
*v*L	+2·6	...	...	...	+0·1	...	+1·3
L	−0·6	+2·9	...	...	...	−1·5	+0·3
*p*L	+0·7	−0·9	+0·1	+0·6	+2·3	...	+0·6
*p*S	−1·5	+0·7	...	+0·4	+0·2	−0·4	−0·1
S	−1·9	+0·8	+1·9	−0·8	...	−0·4	−0·1
*v*S	−3·2	−1·4	...	−1·9	...	...	−2·2

It appears that while there is a considerable drop in the numbers in the *v*B column, this is largely compensated for by the reverse effect in the faint columns. If the blanks could be filled up, it is probable the compensation would be better still.

(4) Taking the means given in Table II. as they stand, it would perhaps be of interest to apply them as corrections to the figures in Table I. of my first paper. Omitting the two extremes, viz., *v*L and *v*S, and taking the means of the groups L and *p*L and *p*S and S, we have:—

L and *p*L	+0·4
*p*S and S	−0·1
Difference	0·5

As the measures are equivalent to stellar magnitudes, this difference in brightness is equivalent to $2{\cdot}5^{0{\cdot}5}$. Hence the mean brightness of the group L and *p*L, derived from the Catalogue, has to be divided by 1·6 to render it comparable with that of the group *p*S and S.

(5) In my former paper the scale for brightness used was 1 to 9. This was, of course, quite arbitrary. From the smoothed means given above we may determine the relative brightness of each order as below:—

	*v*B.	B.	*c*B.	*p*B.	*p*F.	F.	*v*F.	*c*F
Smoothed means	7·4	8·2	9·1	9·9	10·7	11·6	12·4	13·2
Absolute ratios	200	100	44	20	10	4·5	2	1

All we require to know now is the proportion of each order of brightness in the two groups. This was not noted in the original investigation, and I have therefore gone through one hour of R.A. in the catalogue again. I have chosen hour XI for the purpose, as this hour resembles most nearly the mean of the 24 (see Table II. of first paper). The results are given below:—

TABLE III.

A Group, L and *p*L (mean diameter about 2½′).

	6 objects marked		*v*B	multiplied by		200 =	1200
	7	,,	B	,,	,,	100 =	700
	11	,,	*c*B	,,	,,	44 =	484
	10	,,	*p*B	,,	,,	20 =	200
	7	,,	*p*F	,,	,,	10 =	70
	16	,,	F	,,	,,	4·5 =	72
	8	,,	*v*F	,,	,,	2 =	16
	15	,,	*e*F	,,	,,	1 =	15
Total,	80 objects.					Total,	2757
						Mean brightness,	34·5
						Divided by 1·6 to correct systematic error,	21·5

B Group, *p*S and S (mean diameter about 30″).

	5 objects marked		B	multiplied by		100 =	500
	4	,,	*c*B	,,	,,	44 =	176
	13	,,	*p*B	,,	,,	20 =	260
	19	,,	*p*F	,,	,,	10 =	190
	52	,,	F	,,	,,	4·5 =	234
	38	,,	*v*F	,,	,,	2 =	76
	35	,,	*e*F	,,	,,	1 =	35
Total,	166					Total,	1471
						Mean brightness,	8·6

(No correction for systematic error.)

(6) We may conclude from the above table that after allowing for the systematic errors in the catalogue, there is still a marked difference in the brightness of the two groups, which we may attribute to absorption. From the fewness of the photometric observations used, we still cannot regard the figures as reliable; but they are probably not far out, and, as mentioned above, they are more likely to be *over* corrected. The figures should encourage the making of a comprehensive series of measures, either visually or photographically—a work which would be of the greatest value, not only for the elucidation of this particular problem, but for general research on the nebulæ.

Section II.—A Spherical Harmonic Analysis of the Star Distribution.

(7) It was pointed out in the former paper that there is a considerable variation in the brightness of the nebulæ in different parts of the sky. An examination of the star density in those parts of the sky where the greatest differences were noticed appeared to indicate that the stars were subject to the same variations (when freed from galactic effects). Hence it was concluded that this was probably an absorption effect.

(8) In a paper by Professor Turner and the writer (*M.N.*, Jan.

1912) the figures for the brightness of the nebulæ over the whole sky were reduced to spherical harmonics. The agreement between the stars and nebulæ suggested the utility of a similar analysis of the star distribution.

(9) The difficulty is to disentangle the particular variations we are in search of from the much larger effects due to the galactic structure. It was at first hoped that the simpler harmonics would suffice to represent the distribution in relation to the galaxy, and that the residuals would represent any possible irregularities due to other causes, such as those suggested by variations in the brightness of the nebulæ. It was found, however, that the first three orders do not suffice to represent the galactic structure. The computed values in the galaxy itself and near its poles are too small, while the intermediate zones are too high. This supports the idea which has been frequently expressed, that the Milky Way is actually a ring of stars round our system, not merely the effect of looking through a thicker stratum of stars. Although, by employing higher orders, we might eventually represent the observations, we should then be unable to distinguish the harmonics representing the general structure from those irregularities we desire to find. The axes would be so numerous that their approximate coincidence with those found from the nebulæ would carry no weight.

(10) The method was then tried of arranging the material in zones of galactic latitude each 20° in breadth, making nine in all. The position of the galactic pole has been taken as R.A. $12^h\ 50^m$ N. Decl. 26°. Each zone is divided into twelve areas 30° wide, the zero point being that at which the zero of right ascension crosses the galactic equator, viz., $0^h + 64°$. In Table IV. the mean star density (derived from the counts of the two Herschels) is tabulated for each area, and these numbers are expressed in Table V. as ratios to the mean of the pair of zones of equal galactic latitude. In this way the effects of the galaxy are eliminated.

TABLE IV.

G.N.P.D.=	10°.	30°.	50°.	70°.	90°.	110°.	130°.	150°.	170°.
ϕ= 15	6·4	8·0	16·0	50·0	94·0	49·5	14·8	9·0	4·0
45	5·8	10·0	14·3	38·0	78·8	16·3	9·1	6·4	4·6
75	5·3	6·6	11·6	36·5	85·0	25·6	8·5	6·3	5·2
105	4·8	6·6	9·3	37·1	63·0	22·6	12·1	8 9	5·9
135	4·1	4·7	9·4	31·1	63·3	24·7	9·2	8·1	8·2
165	3·4	5·8	11·7	30·9	66·9	22·5	12·8	8·8	4·8
195	3·8	6·5	13·7	37·2	61·9	25·7	11·6	9·9	5·0
225	5·5	5·5	14·9	27·5	60·8	29·5	13·2	8·8	8·5
255	5·6	6·5	13·7	22·8	69·0	31·4	13·7	6·9	7·8
285	5·8	8·6	14·4	50·3	287·5	50·8	11·8	7·2	5·6
315	6·6	8·6	16·7	60·0	208·7	61·7	19·1	7·4	8·6
345	6·5	8·3	13·8	53·1	181 0	39·2	14·7	6·9	3·3
Means of each pair of zones.	5·6	7·5	12·9	37·4	108·0	37·4	12·9	7·5	5·6

TABLE V.

G.N.P.D.=	10°.	30°.	50°.	70°.	90°.	110°.	130°.	150°.	170°.
ϕ= 15	114	107	124	133	88	132	115	120	71
45	104	133	111	102	73	48	71	85	82
75	95	88	90	97	79	69	66	84	91
105	85	88	72	99	59	60	94	119	105
135	73	63	73	83	59	66	71	108	146
165	61	77	90	83	62	60	99	117	86
195	68	87	106	100	57	70	90	132	89
225	98	73	116	74	56	79	102	117	151
255	100	87	106	61	64	84	106	92	139
285	104	115	112	134	266	135	91	96	100
315	118	115	129	226	193	165	148	99	154
345	116	111	107	145	168	105	116	92	59
Means	94	95	103	111	100	89	97	105	106

(11) The variations in Table V. are generally about the same as those of the nebulæ, excepting in the regions about the point $\phi = 285°$, $\theta = 90°$, G.N.P.D., where the stars are exceptionally numerous. In spite of this general agreement, the first and second order harmonics do not agree with those calculated from the nebulæ, the effect of this large dense cluster in the Milky Way being to divert the major axis towards it. To avoid the disturbing effects produced by the irregularities in the Milky Way (which are clearly real clusters), a set of harmonics have been calculated on the basis of the six non-galactic zones only. In order to effect a direct comparison, the nebulæ have been treated in the same way, *i.e.*, the numbers in Table III. of the former paper have been transferred to galactic latitude and longitude (as in Table VI.) and the harmonics calculated from the six non-galactic zones. The agreement is then good.

TABLE VI.

Brightness of the Nebulæ arranged in Zones of Galactic North Polar Distance.

θ	ϕ= 15	45	75	105	135	165	195	225	255	285	315	345	Mean.
10	120	120	120	122	122	122	109	98	98	108	120	120	115
30	120	124	116	105	103	123	105	89	89	87	104	116	107
50	124	117	100	114	89	101	109	97	85	74	95	91	100
70	104	101	88	84	94	117	134	115	114	89	79	82	100
90	96	90	87	90	103	121	104	101	121	101	96	90	100
110	99	89	94	88	121	116	98	111	101	99	84	90	100
130	87	81	88	92	136	126	116	128	97	82	75	84	100
150	83	81	80	95	135	83	109	118	93	82	75	81	93
170	81	81	81	84	89	89	89	102	88	72	72	82	84

Another way out of the difficulty is to calculate harmonics based on the southern hemisphere alone. It should be pointed out that in thus dealing with portions of the sphere only, we are only able to compute the second order harmonics; the first and third orders cannot be calculated in this way.

(12) The second order harmonics may now be collected for comparison. The various coefficients are grouped in Table VII. A (extracted from the paper in *M.N.*, Jan. 1912) and E (Stars, southern hemisphere) are in terms of ordinary R.A. and declination; the others are in galactic latitude and longitude.

TABLE VII.

Second Order Harmonics.

		$(3\cos 2\theta + 1)$.	$\sin 2\theta \cos \phi$.	$\sin 2\theta \sin \phi$.	$\sin^2\theta \cos 2\phi$.	$\sin^2\theta \sin 2\phi$.
A	Nebulæ, whole sky .	+10·0	− 4·0	+ 8·0	−11·0	+ 6·0
B	Stars, whole sky .	− 0·3	+ 8·2	+ 2·8	+ 2·2	26·2
C	Stars (six non-galactic zones only)	− 0·3	+ 8·2	+ 0·1	+14·7	− 0·2
D	Nebulæ as in C . .	+ 0·4	+13·8	+ 4·7	+14·4	+ 6·5
E	Stars, southern hemisphere only	− 0·1	− 0·6	+ 7·0	− 1·8	− 8·0

In the next table (Table VIII.) the position is given of one end of each of the three axes of each ellipsoid. The galactic latitudes and longitudes have been retained—A and E being converted. The means at the foot of each column are obtained from the formulæ

$$\cos\theta_0 = \frac{M_1}{\sqrt{M_1^2 + M_2^2 + M_3^2}} \text{ and } \tan\phi_0 = \frac{M_3}{M_2},$$

where M_1, M_2, and M_3 are the means of the groups $\cos\theta$, $\sin\theta\cos\phi$, and $\sin\theta\sin\phi$, formed from the individual values of θ and ϕ.

TABLE VIII.

Axes of the Ellipsoids.

	Major axis.		Minor axis.		Third axis.	
	θ	ϕ	θ	ϕ	θ	ϕ
	°	°	°	°	°	°
A	66	14	41	136	52	276
B	82	318	71	218	156	261
C	67	0	90	90	24	180
D	58	14	82	99	34	174
E	63	77	30	282	104	351
Means,	62	10	45	125	48	270

It will be noticed that the means fall very close to A in spite of the individual discordances. There is also good agreement between C and D. Another point worth noting is that nearly all the axes show a marked preference for intermediate zones of galactic latitude.

(13) The close accordance between the means and the results originally found for the nebulæ suggests that the causes which produce variations in the brightness of the nebulæ affect the numbers of the stars in the same manner. In such work as this we cannot, of course, entirely ignore the possibility of a chance agreement, but the likelihood that both sets of axes should so nearly coincide is very remote unless from a common cause.

I should like to express my indebtedness to Professor Turner for valuable advice and suggestions in connection with the foregoing.

Section III.—The Hypothetical Distribution of Absorbing Material.

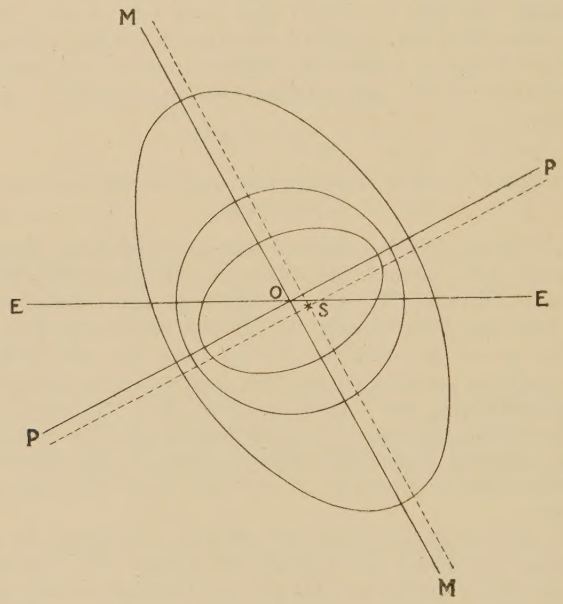
(14) In the previous section reference has been made to the preference of the second order harmonics for the intermediate zones of galactic latitude. If this depended on the nebulæ alone, we might not accept the results with much confidence, but the fact that the use of the material derived from the star distribution gives us similar results induces us to look for an explanation.

(15) Although we cannot be sure that the axes have any relation to the general galactic structure, yet this structure has such an intimate connection with so many of the phenomena of our system that it is natural to look for some connection in this case also.

(16) Let us consider the effect produced by absorbing material distributed uniformly throughout our system, and conforming to its shape, which for the present we may consider as elliptical, with its minor axis coincident with the poles of the galaxy. In this case it is clear that two of the axes of the second order must fall in the plane of the galaxy and the third perpendicular to it. This will still be the case in whatever part of the ellipsoid the Earth (or Sun) is situated. Thus the harmonics cannot be explained in this way.

(17) A possible solution is suggested by Professor Turner's investigations of the direction of the centre of our system (*M.N.*, April 1912). The displacement of the solar system towards 270° R.A. fixes the minor axis (corresponding to the major axis of the harmonics) in the plane at right angles to the line joining the Sun and the centre, but the axis is still parallel to the polar axis of the galaxy. If, however, we introduce a small ellipsoid of absorbing matter with its major axis passing through the poles of the galaxy, we divert the minor axis towards the plane of the galaxy to an extent depending on the relative densities of the two ellipsoids. In the figure the system is represented as it would appear from a view-

point outside the system, in the direction R.A. 19^h Decl. 0°, where the planes of the galaxy and of the Earth's equator cut one another. The medial line of the galaxy is therefore represented by the straight line MM. The solar system appears in the position S, being away from the centre O in the direction 18^h, −12°, and is thus slightly out of the plane of the galaxy (which is actually about 8° nearer the southern galactic pole than the northern). Under these conditions the short axis will appear to be in a direction midway between the two axes MM and PP.



System viewed from Direction of 19^h R.A. +0° Decl.

O, Centre of System; S, Position of Sun; MM, Milky Way; PP, Galactic Poles; EE, Equator.

(18) The figure is of course only a rough approximation. A more likely form would be a gradual merging of one ellipsoid into the other through the spherical form with increasing density. It is interesting to note that there is an example of both types in the published photographs of nebulæ. A Crossley photograph of the nebula H IV. 27, Hydra (N.G.C. 3242), clearly shows two ellipsoids, the major axes of which are separated by 85°, each ellipsoid being distinct and separate. An examination of the fine Mount Wilson photograph of the Andromeda nebula shows that there is a small elliptic condensation at the centre, with a major axis inclined at an angle of 60° to the general axis of the nebula. In citing these cases, it must not be supposed that it is desired to trace a very

close connection between the bright nebulosity in these objects and the dark matter, the possible existence of which we are considering. At the same time it is not unreasonable to suppose that some law of distribution may exist that may affect the particles in both systems in the same way.

(19) To test the utility of the figure in explaining the variable brightness of the stars and nebulæ, it would be interesting to compare the simple harmonics calculated from the figure with the observed values. I have therefore calculated both sets to the sixth order, and they are collected in Table X. The observed values are combined from the numbers in Tables V. and VI. at the points $\phi = 15°, 365°, 165°$, and $195°$, thus embracing the complete circle at $0°$ and $180°$.

The numbers on which the computed values are based are derived by the simple process of measuring the distance from S through the three figures A, B, and C, at $10°$, $30°$, $50°$, etc. We have then to find what values of A, B, and C produce accordance for the axes of the second order. The values $A = 6$, $B = 2{\cdot}5$, $C = 1$, form one set of values that will suit. In Table IX. the values for O and C are tabulated; in Table X. the harmonics computed from them; and in Table XI. the direction of the axes indicated by the harmonics.

Table IX.

Observed and Computed Values along 0° and 180° Galactic Longitude.

10	30	50	70	90	110	130	150	170	190	210	230	250	270	290	310	330	350
117	113	111	116	110	106	100	94	73	88	110	108	86	86	108	101	98	90
129	134	136	150	161	159	157	164	167	156	139	130	129	125	119	114	118	126

Table X.

*Harmonics calculated from Table IX.**

os θ.	Sin θ.	Cos 2θ.	Sin 2θ.	Cos 3θ.	Sin 3θ.	Cos 4θ.	Sin 4θ.	Cos 5θ.	Sin 5θ.	Cos 6θ.	Sin 6θ.
7·2	+ 5·9	− 3·8	+ 5·7	+ 0·9	− 3·6	− 5·8	+ 6·8	+ 6·0	+ 2·8	− 1·0	+ 1·3
15·7	+ 16·9	+ 1·6	− 1·7	− 1·2	+ 0·7	+ 4·6	− 1·4	− 0·8	+ 1·3	− 0·6	− 0·2

Table XI.

Direction of the Axes.

	θ.	2θ.	3θ.	4θ.	5θ.	6θ.
O	39°	62°	95°	33°	77°	21°
C	313	66	110	41	60	3
O − C	+ 86	− 4	− 15	− 8	+ 17	+ 18
Maximum O − C	180	90	60	45	36	30

The agreement down to the fourth order is very good; the fifth

* In studying these harmonics it should be remembered that the observed and computed values are opposite in sign, since *minimum* depth of absorbing material corresponds to *maximum* brightness of the objects seen through it.

and sixth are not so good, although the deviation is considerably less than the maximum.

(20) As a further test, we will compare theory and observation, taking as our view-point the direction of the north galactic pole. To agree with the former figure, S must be in the direction 258° galactic longitude. Our difficulty is that we have no direct means of determining whether the system from this view-point is circular or elliptic, and, if elliptic, the direction of the axes. We may obtain a rough idea from the breadth of the Milky Way at various points. The calculated second order harmonics, from measures of its breadth, indicate considerable ellipticity, the breadth at 317° (the minor axis) working out at 14·8°, the mean being 12° and the breadth in the direction of the major axis 9·2°. These rough dimensions have been adopted, and O and C for this zone, using the same values for A, B, and C as before, are then as in Table XII.

TABLE XII.

Observed and Computed Values along the Galactic Equator.

	$\varphi =$	15	45	75	105	135	165	195	225	255	285	315	345
O		96	90	87	90	103	121	104	101	121	101	96	90
C		182	246	269	239	193	143	116	103	85	79	90	122

In this case it was necessary to use the nebulæ alone for the observed values, as the stars are distributed so irregularly in the Milky Way. The harmonics are given in Table XIII.

TABLE XIII.

Harmonics for Numbers in Table XII.

	Cos θ.	Sin θ.	Cos 2θ.	Sin 2θ.	Cos 3θ.	Sin 3θ.	Cos 4θ.	Sin 4θ.	Cos 5θ.	Sin 5θ.
O	− 6·5	−6 8	+ 1·7	− 0·8	+0·2	+7·5	+2·5	−4·1	−7·5	+2·3
C	+12·8	+8·5	−15·8	+16·7	−2·3	−1·3	−2·5	−0·4	+0·7	−0·8

The corresponding axes are given in Table XIV.

TABLE XIV.

	θ.	2θ.	3θ.	4θ.	5θ.
O	227	167	29	76	33
C	214	157	10	92	26
O − C	+ 15	+ 10	+19	− 16	+ 7
Maximum O − C	180	90	60	45	36

The agreement here is even better than in the case of the other view-point.

(21) In default of definite information as to the causes that would be likely to produce a change from one ellipsoid into another as outlined in § 17, this hypothesis can only be regarded as tentative. At the same time the excellent agreement between the observed and computed brightnesses is a point in its favour, and entitles it to some consideration pending further information.